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Time-dependent natural convection of micropolar fluid in a wavy triangular cavity

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ABSTRACT

Natural convection of micropolar fluid in a right-angled wavy triangular cavity has been analyzed numerically. Governing equations formulated in dimensionless stream function, vorticity and temperature using the Boussinesq and Eringen approaches with appropriate initial and boundary conditions have been solved by finite difference method of the second-order accuracy. The effects of the dimensionless time, Prandtl number, vortex viscosity parameter, and undulation number on streamlines, isotherms, vorticity isolines as well as average Nusselt number at wavy wall and fluid flow rate inside the cavity have been studied. Obtained results have revealed essential heat transfer reduction and fluid flow attenuation with vortex viscosity parameter.

1. Introduction

The theory of simple microfluids, as developed by Eringen [1], has been a field of active research for the last few decades due to many practical applications. In the last few decades, the research interest in micropolar fluid theory has significantly increased due to its enormous applications in many industrial processes, such as, flow of low concentration suspensions, liquid crystals, animal blood, lubrication, colloidal suspensions, turbulent shear flows, etc. (see Wang and Chen [2]). This theory can be also used to explain the experimentally observed phenomenon of the drag reduction near a rigid body in fluid containing small amounts of additives when compared with the skin friction coefficient in the same fluids without additives (see Wang and Chen [2]). These fluids cannot be explained on the basis of Newtonian fluid flow theory. Micropolar fluids represent a subclass of microfluids and were also introduced by Eringen [3]. These fluids display the effects of local rotary inertia and couple stresses that can be used to explain the flow features of the above-mentioned fluids for which the classical theory of Newtonian fluids is inadequate. The theory of micropolar fluids was extended subsequently by Eringen [4] to include thermal effects, to the so-called thermomicropolar fluids. Since Eringen [1] has published the micropolar fluid theory, many authors have investigated various flow and heat transfer problems. The common comment in respect of micropolar fluids was that there are no experiments whatsoever in which any of the material moduli could be measured. Hoyt and Fabula [5] has shown experimentally that the fluids containing minute polymeric additives indicate considerable reduction of the skin friction (about 25–30%), a concept which can be well explained by the theory of micropolar fluids. Power [6] has shown that the fluid flowing in brain (Cerebrospinal fluid) is adequately modeled by micropolar fluids. The works of Migun [7] and Kolpashchikov et al. [8] demonstrated an experimental method of determining parameters characterizing the microstructure of such fluids and seems to have laid to rest many unanswered questions on the theory. Extensive reviews of the theory and applications can be found in the review articles by Ariman et al. [9,10] and the recent books by Łukaszewicz [11] and Eringen [12]. For more recent developments in the field of non-Newtonian fluid dynamics see Hayat and Hutter [13], Rajagopal and Srinivasa [14], Fetecau et al. [15], Lok et al. [16], Magyari et al. [17], Gibanov et al. [18] as well as references therein. We also mention here that the boundary layer theory of micropolar fluids were founded by Peddieson and McNitt [19], and Wilson [20].

It is worth mentioning that an important contribution in micropolar flow dynamics was made by Sankara and Watson [21], when they investigated the flow of micropolar fluids past a

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Nomenclature

A	aspect ratio parameter
a	wavy contraction ratio
b	shape parameter
Da	Darcy number
g	gravitational acceleration ($m \cdot s^{-2}$)
H	height of the cavity (m)
j	micro-inertia density (m^2)
K	dimensionless micropolar parameter or dimensionless vortex viscosity parameter
L	bottom wall length (m)
$\mathbf{N}$	microrotation vector (s^{-1})
N	dimensionless microrotation vector component
$\overline{N}$	dimensional microrotation vector component (s^{-1})
$\mathbf{n}$	normal unit vector
n	dimensionless micro-gyration parameter
Nu	local Nusselt number
$\overline{Nu}$	average Nusselt number
$\bar{p}$	dimensional pressure (Pa)
Pr	Prandtl number
Ra	Rayleigh number
S	length of the wavy wall (m)
T	dimensional temperature of the fluid (K)
t	dimensional time (s)
T_c	dimensional temperature of the cooled wall (K)
T_h	dimensional temperature of the hot wall (K)
u, v	dimensionless velocity components along the axes x and y, respectively
$\bar{u}$, $\bar{v}$	dimensional velocity components along the axes $\bar{x}, \bar{y}$ ($m \cdot s^{-1}$)
$\mathbf{V}$	dimensional velocity vector ($m \cdot s^{-1}$)
$\bar{x}$	dimensional coordinate measured along the bottom wall of the cavity (m)
$\bar{x}_1$	dimensional location of the left wavy wall (m)
$\bar{x}_2$	dimensional location of the right inclined wall (m)
$\bar{y}$	dimensionless coordinate measured along the vertical wall (m)
x, y	dimensionless Cartesian coordinates

Greek symbols

α	thermal diffusivity ($m^2 \cdot s^{-1}$)
β	volumetric expansion coefficient of the fluid (K^{-1})
γ	spin-gradient viscosity ($kg \cdot m \cdot s^{-1}$)
$\Delta\tau$	dimensionless time step
δ_θ	thickness of the thermal boundary layer (m)
δ_v	thickness of the momentum boundary layers (m)
θ	dimensionless temperature
κ	dimensional micro-rotation or dimensional vortex viscosity (Pa·s)
λ	undulation number
μ	dynamic viscosity (Pa·s)
ν	kinematic viscosity ($m^2 \cdot s^{-1}$)
ξ, η	new independent dimensionless variables
ρ	fluid density ($kg \cdot m^{-3}$)
$\sigma(y)$	wavy wall equation
τ	dimensionless time
ψ	dimensionless stream function
ω	dimensionless vorticity

stretching sheet. Heruska et al. [22] extended the work of Sankara and Watson [21] by considering the mass suction or injection through the porous sheet. Hassanien and Gorla [23] explained the heat transfer in a micropolar flow over a non-isothermal stretching sheet with suction and blowing. Na and Pop [24] also considered the boundary layer flow of micropolar fluid due to continuously stretching boundary. Also, Ishak et al. [25] have investigated the heat transfer over a stretching surface with variable heat flux in micropolar fluids.

On the other hand, it should be mentioned that wavy geometries are used in many engineering systems as a means of enhancing the transport performance (see the papers by Chiu and Chou [26] Chu et al. [27], Wang and Chen [2], Mahmud and Fraser [28], Chen and Cho [29], Al-Amiri et al. [30], Sheremet et al. [31–33] and the book by Shenoy et al. [34]).

The main objective of the present study is to analyze numerically the natural convection of micropolar fluid in a right-angled wavy triangular cavity. The governing equations formulated in dimensionless stream function, vorticity and temperature using the Boussinesq approximation and Eringen's approach [1,3,4] have been solved by finite difference method. The effects of the dimensionless time, Prandtl number, vortex viscosity parameter, and undulation number on streamlines, isotherms, vorticity isolines as well as average Nusselt number at wavy wall and fluid flow rate have been studied. To the best of our knowledge, this problem has not been studied before. Thus, the results are new and original. On the other hand, it should be pointed out that the Navier–Stokes equations which govern fluid flow problems must be solved numerically in most of time. Exact solutions can be obtained for small parts of practical problems because most of these problems are complex. Numerical simulation of incompressible fluid flow problems in complex geometries is a computational challenge.

We mention to this end that natural convection heat transfer in cavities is importance to many engineering systems, such as solar energy collectors, building energy components, cooling of electrical units, and the heat preservation of thermal transport circuits. To date, there are many studies involving natural convection in a square cavity, and less work has been done on triangular-type enclosures. In solar heating for example, many geometric configurations might be considered based on the heater location on the side walls of the square, triangular-type, and so on. Moreover, study of natural convection in triangular enclosures can be found in the design of building roofs and attics, solar energy collectors, cooling of electronical devices as PC, TV etc. It arises in these geometries due to temperature difference between inside heating and environmental conditions. Natural convection in triangular enclosures under different thermal boundary conditions in non-porous media using air as working fluid has been extensively studied in the past years (see Varol et al. [35]).

2. Mathematical formulation

Fig. 1 shows the considered right-angled triangular cavity filled with a micropolar fluid. The domain of interest is bounded by bottom adiabatic wall of length L, left isothermal wavy wall of vertical length H with high temperature T_h and right inclined isothermal wall with low temperature T_c. All walls of the cavity are supposed to be rigid and impermeable. It is considered that the left wavy wall and right inclined flat wall of the cavity are described by the relations $\bar{x}_1 = L - L[a + b\cos(2\pi\lambda\bar{y}/H)]$ and $\bar{x}_2 = L(1 - \bar{y}/H)$, respectively, where $a + b = 1$.

The physical properties of the micropolar fluid are supposed to be constant except for the density in the buoyancy force term of