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The least action principle for heat conduction and its optimization application

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ABSTRACT

The least action principle is used in various disciplines including linear transport processes. However, non-equilibrium thermodynamic analyses of linear transport processes involve the dot product of the thermodynamic flux and the thermodynamic force which must be the entropy production rate in the linear phenomenological law for such processes; thus, Fourier's heat conduction law cannot be derived based on the variation of the entropy production rate. A generalized linear phenomenological law for various types of linear transport processes, including heat conduction, mass diffusion, electric conduction and fluid flow in porous medium is introduced here where the dot product of a generalized flux and a generalized force is taken as the action to give the generalized linear phenomenological law. For heat conduction, the entransy dissipation rate, which is the dot product of the heat flux and the negative of the temperature gradient, is taken as the action, and the variation of the entransy dissipation rate then leads to Fourier's heat conduction law with constant thermal conductivity. Hence, the action of heat conduction process is the entransy dissipation rate rather than the entropy production rate. A nonlinear constitutive relation for the heat conduction with temperature dependent thermal conductivity is then converted to a linear problem by introducing a generalized temperature, which gives a least generalized entransy dissipation principle for nonlinear heat conduction processes. Finally, the least entransy dissipation principle is applied to optimize a one-dimensional heat conduction problem without heat-work conversion as an example where the minimum entropy generation principle is not applicable.

1. Introduction

The least action principle states that the path (or distribution) actually followed by a physical system is that for which the action is minimized [1]. The action is usually defined by an integral, with the differential equations describing the physical phenomenon derived based on the extremum of the corresponding actions. Historically, the first such principle was Fermat's principle in optics, which claimed that light travels between two given points along the path having the shortest time. Many other least action principles have been proposed in various fields including mechanics, electrical phenomena and thermodynamics. The least action principles give additional insight into the physical phenomenon and have been widely used in engineering [2]. This principle is probably a universal natural law as the great mathematician Euler said [3]:

There is consequently, no doubt but all the effects of the world can be derived by the method of maxima and minima from their final causes as well as from their efficient ones.

In addition, the least action principles provide as a simple alternative formulation of the differential equations governing various phenomena and can be used to develop the efficient numerical methods [3–7].

Many researchers have sought a least action principle for the known phenomenological laws in non-equilibrium thermodynamics. In 1931, Onsager [8,9] proposed the principle of the least energy dissipation for a general linear non-equilibrium process. The minimum entropy production principle by Prigogine [10,11] is one of the best known examples, which states that the steady state of a linear non-equilibrium process should correspond to the minimum rate of entropy production. Strictly speaking, the steady state of a linear non-equilibrium process will lead to the maximum entropy production rate for prescribed thermodynamic forces, while the minimum entropy production principle is valid only for cases with prescribed fluxes [12,13]. Therefore, a rigorous statement should be that the steady state of a linear non-equilibrium process corresponds to the extremum entropy production rate. de Groot et al. [14] extended the entropy production minimization principle by taking spatial variations into account. Additionally, Gyarmati [15] proposed a variational principle for the fundamental laws of the

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Nomenclature

$\mathbf{J}_t$	thermodynamic force
$\mathbf{F}_t$	thermodynamic flux
$\mathbf{L}$	phenomenological coefficient matrix
T	temperature
$\mathbf{q}$	heat flux
l_{qq}	phenomenological coefficient
k	thermal conductivity
T	temperature
$\mathbf{J}$	generalized flux
$\mathbf{F}$	generalized force
g	entransy dissipation rate
Q	overall heat transfer rate
F	generalized temperature
R_G	entransy dissipation-based thermal resistance
D	length of plate heat exchanger
H	width of plate heat exchanger
$w(y)$	thickness distribution of plate heat exchanger
W	cross sectional area of plate heat exchanger
$\dot{S}$	total entropy production rate
$\dot{G}$	total entransy dissipation rate
Greek symbols	
σ_S	entropy production rate
σ_{Sw}	weighted entropy production rate
$\Phi_\mathbf{J}$	dissipation function
Γ	intensive variable
θ	dimensionless temperature
ξ	dimensionless abscissa
Subscripts	
0	reference state
ge	generalized

dissipative processes both in the differential (local) and in integral (global) forms. However, they found that the constitutive relation between the temperature gradient and the heat flux derived based on entropy production-based principles should have the thermal conductivity inversely proportional to the square of the temperature [16–18]. Since there is not known material whose thermal conductivity obeys this relation, there is no real situation involving heat conduction where the steady state can be predicted quantitatively by minimizing the entropy production rate. Glansdorff et al. [11] then had to modify the expression for the entropy production rate to recover the exact constitutive relation for the heat conduction.

Moreover, the least action principle can be further used to optimize a problem for given constraints. For example, some researchers have extended the entropy production-based principle to heat transfer optimization analyses. Bejan [19] developed the entropy generation minimization principle (EGMP) called thermodynamic optimization that the best heat transfer rate corresponds to the minimum entropy generation. A series of studies have used the EGMP concept to optimize heat transfer processes [20–27]. However, some counter examples have been identified where EGMP does not give the best heat transfer performance [28–30]; thus, the minimum entropy production rate does not always optimize the heat transfer process. For example, minimizing the entropy production rate for the volume-point heat conduction problem with constant thermal properties does not give the lowest average temperature in the domain for a given heat source [28]. Shah and Skiepko [29] analyzed the relationship between the effectiveness and the entropy generation in eighteen different types of heat exchangers and found that the effectiveness is maximized in some cases, minimized in some cases, or any value in between in some cases when the entropy production rate is minimized. Chen, et al. [30] pointed out that heat transfer processes should be classified into two categories according to their purposes as heat-work conversion in thermodynamic cycles and purely object heating or cooling. For the first category, the entropy generation should be used to optimize the heat transfer processes, so this is called thermodynamic optimization. For the second category, the entransy dissipation should be used to optimize the heat transfer processes, so these are called heat transfer optimization [30]. Hence, it is necessary to distinguish the least action principle for such two categories of heat conduction.

Examinations in this paper start from generalized linear transport processes, such as heat conduction and mass diffusion. Analyses show that the dot product of the generalized flux and the generalized force in the phenomenological law should be its action, whose variation can recover the constitutive relation of transport process. A least action principle based on the entransy dissipation is then given which can recover Fourier's heat conduction law with constant thermal conductivity. Furthermore, this principle is extended to cases with temperature-dependent thermal conductivity by defining a generalized temperature with a least generalized entransy dissipation principle. Finally, optimization principles are given based on the entransy dissipation rate and the generalized entransy dissipation rate with a one-dimensional heat conduction problem given as an example.

2. Least action principle for linear phenomenological law

2.1. Entropy production-based least action principles

Entropy production rate has been used in non-equilibrium thermodynamics to construct least action principles by using the variation of the entropy production rate to derive linear phenomenological laws for transport processes. In this theory, an important restriction is that the dot product of the thermodynamic flux, $\mathbf{J}_t$, and the thermodynamic force, $\mathbf{F}_t$, should be the entropy production rate [11,14,31],

$$\sigma_S = \mathbf{F}_t \cdot \mathbf{J}_t. \tag{1}$$

To avoid arbitrary choices of the thermodynamic flux and force, Onsager [8,9] proposed that the flux should be the time derivative of the extensive state variable which characterizes the displacement from thermodynamic equilibrium. A dissipation function was then introduced by analogy to the Rayleigh dissipation function by Onsager [8],

$$\Phi_\mathbf{J} = \frac{1}{2}\mathbf{J}_t \cdot \mathbf{L}^{-1} \cdot \mathbf{J}_t, \tag{2}$$

where $\mathbf{L}$ is a phenomenological coefficient matrix which should be constant. The combination of the entropy production rate and the dissipation function yields the least dissipation of energy principle [8],

$$\frac{\delta}{\delta \mathbf{J}_t}[\sigma_S - \Phi_\mathbf{J}] = 0 \Rightarrow \mathbf{J}_t = \mathbf{L} \cdot \mathbf{F}_t, \tag{3}$$

which gives the constitutive relation between the prescribed thermodynamic flux and the thermodynamic force.