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A simplified thermal lattice Boltzmann method without evolution of distribution functions

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ABSTRACT

In this paper, a simplified thermal lattice Boltzmann method (STLBM) without evolution of the distribution functions is developed for simulating incompressible thermal flows. With the assistance of the fractional step technique, the macroscopic governing equations recovered from Chapman–Enskog (C–E) expansion analysis are resolved through a predictor–corrector scheme. Then in both the predictor and corrector steps, using the isentropic properties of lattice tensors and relationships of C–E analysis, the macroscopic flow variables are explicitly calculated from the equilibrium and non-equilibrium distribution functions. In STLBM, the equilibrium distribution functions are calculated from the macroscopic variables, while the non-equilibrium distribution functions are evaluated from the differences between two equilibrium distribution functions at different locations and time levels. Therefore, STLBM directly updates the macroscopic variables during the computational process, which lowers the virtual memory cost and facilitates the implementation of physical boundary conditions. Through von Neumann stability analysis, the present method is proven to be unconditionally stable, which is further validated by numerical tests. Three representative examples are presented to demonstrate the robustness of STLBM in practical simulations and its flexibility on different types of meshes and boundaries.

1. Introduction

In recent decades, the lattice Boltzmann method (LBM) has been continuously developed [1–4] and applied to various kinds of problems [5–11]. As a mesoscopic method, LBM offers an alternate approach to study the fluid problems, and is welcome among researchers due to its simplicity and explicitness. Based on kinetic theory, LBM solves the lattice Boltzmann equation (LBE) by tracking the evolution of the distribution function, which consists of two simple processes: "streaming" and "collision". Through Chapman–Enskog (C–E) expansion analysis, LBE can be proven to recover the macroscopic Navier–Stokes (N-S) equations. The relevant macroscopic properties can be easily obtained from the moments of the distribution functions.

Effective simulation of thermal flows has always been a popular topic in Computational Fluid Dynamics (CFD) due to its value both in academic explorations and in engineering applications [12–18]. As an advanced CFD method, the lattice Boltzmann method has also been extended to this topic [19–23]. Unlike isothermal flows, the effects of the temperature/energy are considered in simulating thermal flows. Therefore, various LBM models were proposed to incorporate the evolution of the temperature/energy into the computational process, which can be categorized into three approaches: the multispeed model [24,25], the passive scalar approach [26,27] and the thermal energy distribution model [28–32]. Among these models, the thermal energy distribution model is most recently developed, and has attracted much attention due to its better numerical stability. In this model, the thermal effects are incorporated into the computation by the evolution of the internal energy distribution function. Recently, Peng et al. [33] further simplified the thermal energy distribution model by neglecting the compression effect and heat dissipation. Such simplifications are robust in thermal simulations at the incompressible limit. The thermal energy distribution function model [30,34,35] remains the only method able to simulate incompressible viscous flows with imposed heat flux at the boundary (Neumann boundary condition).

The thermal LBM inherits the merits of the standard isothermal LBM but, at the same time, also suffers from its drawbacks. Firstly, due to the lattice uniformity, the thermal LBM is also only applicable on uniform meshes. Extra computational efforts are required to

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apply it on non-uniform meshes. Secondly, the thermal LBM [28,33] requires large amounts of virtual memory, because the density and the internal energy distribution functions along all lattice velocity directions and at every grid point need to be stored during the computation. Another drawback is that the physical boundary conditions need to be transformed into the conditions of the distribution functions, which is a common problem for methods developed within LBM framework. Such transformations are quite challenging for cases with curved boundaries. In addition, constrained by numerical instability, simulating cases with high Reynolds number or high Rayleigh number is quite challenging for standard LBM [36]. To overcome these shortcomings, Wang et al. [37,38] combined the lattice Boltzmann solver and conventional Navier–Stokes solver, and proposed a novel thermal lattice Boltzmann flux solver (TLBFS). In TLBFS, FVM is applied globally to solve the macroscopic governing equations, while LBE solutions are reconstructed through C–E analysis at each cell interface to recover macroscopic fluxes. One important contribution of TLBFS is its treatment of the non-equilibrium distribution function, which is approximated by the difference between two equilibrium distribution functions at different locations and time levels. This solver has been proven to be both accurate and efficient, with easy implementation of boundary conditions. However, it should be noted that TLBFS involves two sets of solvers, which makes it inconvenient for practical applications. This flaw motivates us to develop the present simplified thermal lattice Boltzmann method which retains the merits of TLBFS but only requires a single solver.

In this paper, we propose a simplified thermal lattice Boltzmann method (STLBM) for simulating incompressible thermal flows. Based on the Chapman–Enskog expansion analysis, the LBE solutions can satisfy the macroscopic governing equations with the second-order of accuracy in space. By applying fractional step method, the recovered macroscopic governing equations can be solved through a predictor–corrector scheme. In STLBM, the formulations in the predictor–corrector steps are reconstructed from lattice properties and the relationships given by C–E analysis, which only involve equilibrium and non-equilibrium distribution functions. Specifically, the equilibrium distribution functions are obtained from the macroscopic flow variables; and the non-equilibrium distribution function is simply approximated by the difference between two equilibrium distribution functions at different locations and time levels. Therefore, unlike the standard thermal lattice Boltzmann method which tracks the evolution of the distribution functions, the present STLBM directly updates the macroscopic variables. Accordingly, without storing the distribution functions, STLBM presents a remarkable saving in virtual memory. Additionally, the physical boundary conditions can be implemented directly. Finally, the simplified lattice Boltzmann method shows nice performance in terms of numerical stability, and can be analytically proven to be unconditionally stable.

The rest of the paper is organized as follows. In Section 2, the basic macroscopic governing equations, the thermal lattice Boltzmann method, and the Chapman–Enskog expansion analysis that links these two systems are briefly introduced. Section 3 gives detailed derivation process, the von Neumann stability analysis and the boundary treatment of the simplified thermal lattice Boltzmann method proposed in this paper. Section 4 presents three representative numerical examples (natural convection in a square cavity, the natural convection in a concentric annulus and the mixed heat transfer from a heated circular cylinder) to validate the convergence, stability, and accuracy of the present method, as well as its adaptability on non-uniform meshes with curved boundaries. The numerical results are compared with reference data in literature. Brief conclusions are finally drawn in Section 5.

2. Macroscopic governing equations and thermal lattice Boltzmann method

2.1. Macroscopic governing equations

For general thermal viscous flows, the macroscopic governing equations include the following continuity, momentum and energy equations [39]

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{u} = 0 \tag{1}$$

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u}\mathbf{u}) = -\nabla p + \upsilon \nabla \cdot \left[\nabla \rho \mathbf{u} + (\nabla \rho \mathbf{u})^T\right] + \mathbf{F}_E \tag{2}$$

$$\frac{\partial}{\partial t}(\rho e) + \nabla \cdot (\rho \mathbf{u} e) = \chi \nabla^2 (\rho e) \tag{3}$$

where ρ, $\mathbf{u}$, p and υ denote the density, the velocity vector, the pressure and the kinematic viscosity, respectively. $\mathbf{F}_E$ is the external force term; χ represents the thermal diffusivity; and e is the internal energy defined as $e = DRT/2$. D, R and T are the number of spatial dimensions, the gas constant and the temperature, respectively. In the present work, we set $D = 2$ and $R = 1$ to maintain consistency with previous studies [37].

The heat transfer can be conducted through conduction, convection and radiation [40]. Here, we mainly focus on the heat convection process in the fluid. Forced convection and natural convection are the two major heat convection problems. For forced convection, the bulk motion of the fluid is driven by external means, e.g., fans or pumps. Therefore, it is usually assumed that the internal energy, or the temperature, has no effect on the velocity field (momentum equation). For natural convection, the motion of the fluid is driven by the buoyancy force, which is related to the temperature difference. Consequently, the momentum and energy equations have to be coupled, and the velocity and the temperature affect each other.

2.2. Thermal lattice Boltzmann method

The thermal lattice Boltzmann model with BGK approximation [28,33] can be expressed as

$$f_\alpha(\mathbf{r} + \mathbf{e}_\alpha \delta_t, t + \delta_t) = f_\alpha(\mathbf{r}, t) + \frac{f_\alpha^{eq}(\mathbf{r}, t) - f_\alpha(\mathbf{r}, t)}{\tau_\upsilon}, \quad \alpha = 0, 1, \ldots, M \tag{4}$$

$$g_\alpha(\mathbf{r} + \mathbf{e}_\alpha \delta_t, t + \delta_t) = g_\alpha(\mathbf{r}, t) + \frac{g_\alpha^{eq}(\mathbf{r}, t) - g_\alpha(\mathbf{r}, t)}{\tau_c}, \quad \alpha = 0, 1, \ldots, N \tag{5}$$

where f_α and g_α are the density distribution function and the internal energy distribution function along the α direction, respectively; The superscript "eq" denotes the equilibrium state; τ_υ and τ_c are single relaxation parameters related to the kinematic viscosity and the thermal diffusivity, respectively; δ_t is the time step applied in the model; M and N are the number of the lattice velocities used in the density and the internal energy distribution functions respectively. In thermal LBM, the macroscopic properties can be obtained from the conservation laws of mass, momentum and energy:

$$\rho = \sum_\alpha f_\alpha, \quad \rho \mathbf{u} = \sum_\alpha f_\alpha \mathbf{e}_\alpha, \quad \rho e = \sum_\alpha g_\alpha \tag{6}$$

The thermal lattice Boltzmann method is associated with the lattice velocity model. In the present work, the 9-bit lattice velocity model (D2Q9) is applied (see Fig. 1) [28,41,42], which consists of the following lattice velocity directions