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Rotating flow of Maxwell fluid with variable thermal conductivity: An application to non-Fourier heat flux theory

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ABSTRACT

In this work we analytically explore the flow and heat transfer of upper-convected Maxwell (UCM) fluid in rotating frame. Fluid with temperature dependent thermal conductivity is considered. A non-Fourier heat flux term, featuring the thermal relaxation effects, is employed to model heat transfer process. Boundary layer approximations are invoked to simplify the governing system of partial differential equations which are later converted to self-similar forms via similarity transformations. Mathematical model comprises of interesting quantities which include the rotation parameter λ, Deborah number β, Prandtl number Pr, dimensionless thermal relaxation time γ and parameter ε. Uniformly convergent approximate series solutions are obtained by means of homotopy analysis method (HAM). Admissible values of the auxiliary parameter in HAM are determined by plotting the so-called $\hbar$-curves. We noticed that hydrodynamic boundary layer becomes thinner due to the inclusion of elastic effects. The rotation parameter λ also serves to reduce the boundary layer thickness. A comparative study of Cattaneo–Christov and Fourier models is also presented and analyzed.

1. Introduction

Non-Newtonian fluid mechanics has been a fascinating and challenging subject as it covers numerous central problems from chemical, petroleum, polymer and food processing industries. Non-Newtonian fluid models are useful to describe the flow behaviors of commonly encountered fluids in nature and industry such as polymeric liquids, biological fluids, motor oils, pastes, slurries and many other complex mixtures. Viscoelastic fluids are special non-Newtonian materials in which applied shear stress is a memory function of the deformation rate. In these fluids, the deformation rate gradually reduces when the shear stress is eliminated. This phenomenon is known as stress relaxation. Moreover the time taken by the fluid for partial elastic recovery upon the removal of stress is relaxation time. One of the frequently applied viscoelastic models is the upper-convected Maxwell model. Researchers have given special focus towards the boundary layer flows of Maxwell fluid in the recent past. Han et al. [1] employed the newly developed Cattaneo–Christov law to model heat transfer effects in Navier-slip flow of Maxwell fluid. Maxwell fluid flow in rotating frame with non-Fourier heat flux formulation was analytically addressed by Mustafa [2]. Numerical treatment for convective flow of Maxwell nanofluid with Brownian diffusion and thermophoresis was made by Mustafa et al. [3]. Mixed convection in the Maxwell fluid flow adjacent to exponentially stretching vertical surface with magnetic field effects was examined by Kumari and Nath [4]. In this study, equations governing the locally similar flow were treated through Chebyshev finite difference scheme. Bhattacharyya et al. [5] determined multiple solutions for Maxwell fluid flow due to porous shrinking sheet using shooting method. Later, Javed and Ghaffari [6] used parallel shooting method to analyze the oblique stagnation-point flow of Maxwell fluid due to stretching surface. Nandy [7] investigated the unsteady flow of Maxwell nanofluid with Navier slip boundary condition. Hayat et al. [8] explored the three-dimensional Maxwell fluid flow with nanoparticles utilizing Brownian motion and thermophoresis effects. Steady flow of Maxwell fluid over permeable surface was explored by Cao et al. [9] using collocation technique equivalent to implicit Runge–Kutta method. Recently a variety of heat transfer problems involving Maxwell fluid have appeared (see [10–13] and Refs. therein).

Heat conduction model developed by Fourier [14] has been widely employed to model heat transfer processes in many practical situations. However this model suffers from a serious drawback as it gives rise to a parabolic energy equation in temperature field which means that transfer of heat will take place instantly once the

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temperature difference is imposed. This contradicts with the well known "Principle of Causality". Keeping this in view, some noteworthy generalizations of the classical Fourier effect have been proposed in the past. For example, the dual-phase-lag-model [15], which accounts for thermal lagging in time or time delay, has been widely applied for the description of micro-scale heat transfer. In the recent past, thermomass model based on the inertia of heat was also applied to explain the non-Fourier aspect in different situations [16–18]. Christov [19] suggested a successful generalization of Fourier law in terms of thermal relaxation effect which is defined as time needed to develop steady-state heat transfer once the temperature gradient is introduced. Straughan [20] investigated convection effects in the horizontal layer of incompressible fluid over a flat plate utilizing Cattaneo–Christov heat flux. Tibullo and Zampoli [21] proved existence and uniqueness of solutions for equations governing heat transfer in incompressible fluids through Cattaneo–Christov theory. Haddad [22] examined thermal instability in Brinkman porous media considering non-Fourier heat flux. Khan et al. [23] modeled and analyzed the exponentially stretched flow of Maxwell fluid with thermal relaxation effects. Hayat et al. [24] considered rotating flows of Jeffrey fluid over a porous stretching sheet considering non-Fourier heat flux theory. Salahuddin et al. [25] also used non-Fourier heat flux law to address the stretched flow of Williamson fluid influenced by magnetic force. Rubab and Mustafa [26] performed an analytical treatment for three-dimensional Maxwell fluid flow and Cattaneo–Christov heat conduction in the existence of Lorentz force. Further attempts in this direction were made by Shehzad et al. [27], Abbasi et al. [28], Sui et al. [29], Li et al. [30] and Liu et al. [31].

To our knowledge, viscoelastic fluid flow in the regimes of rotating frame and variable thermal conductivity has not been explored yet. Thus purpose of present research is three fold. Firstly, to formulate the laminar flow of Maxwell fluid bounded by stretchable surface in rotating frame of reference. Secondly, to analyze heat transfer problem with thermal relaxation effect and temperature-dependent thermal conductivity. Finally, to solve the governing self-similar equations by homotopy analysis method (HAM), introduced by Liao [32]. HAM is considered to be better that conventional perturbation methods as well as non-perturbation methods due to the following reasons. Different from perturbation techniques, there is no requirement of small/large parameter in differential system while applying the HAM. This makes HAM valid for weakly as well as strongly non-linear problems. HAM provides an effective way of adjusting the convergence rate of the series solutions by means of auxiliary parameter $\hbar$. Such flexibility is not available in other non-perturbation approaches such as Adomian decomposition method (ADM), homotopy perturbation method (HPM), variational iteration method (VIM) etc. Additionally, in HAM, there is no restriction on the choice of base functions and auxiliary linear operators. Analytical solution is always handy for scientists and engineers as it gives solutions which are valid for the entire domain. However the numerical methods provide solutions as discrete data which is often time consuming to generate the solution curve. Consequently, we preferred HAM here for finding analytical solution of the present non-linear problem. Convergence analysis is discussed and values of the auxiliary parameter in HAM are obtained by plotting $\hbar$-curves. Influence of parameters on the flow fields is shown graphically. Numerical computations for the missing slopes at the wall are presented for broad range of embedded parameters.

2. Problem formulation

Let us consider the laminar flow of an incompressible Maxwell fluid over a stretchable surface. We choose the Cartesian coordinate system such that the surface is aligned with the xy-plane and fluid is considered in the space $z \geqslant 0$. The surface is assumed to stretch in the x-direction with rate a. Moreover, the fluid rotates continuously about the z-axis with constant angular velocity Ω. Physical sketch of the problem is shown in Fig. 1. The temperature of the sheet, denoted by T_w, is constant and assumed to be greater than the ambient temperature T_∞. We take into account non-Fourier heat conduction model, proposed by Christov [14], to inspect the heat transfer characteristics. Thus relevant equations embodying the Maxwell fluid flow in rotating frame are presented below:

$$\nabla \cdot \mathbf{V} = 0, \tag{1}$$

$$\rho[(\mathbf{V}\cdot\nabla)\mathbf{V} + (\mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{r})) + (2\mathbf{\Omega}\times\mathbf{V})] = -\nabla p + \nabla\cdot\mathbf{S}, \tag{2}$$

where ρ is the fluid density, p the pressure and $\mathbf{\Omega} = [0, 0, \Omega]$ the angular velocity. The term $(2\mathbf{\Omega}\times\mathbf{V})$ is the Coriolis acceleration while the term $(\mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{r})) = -\nabla(\Omega^2 r^2/2)$ represents the centrifugal acceleration which is being balanced by the pressure gradient $-\nabla p$. The extra stress tensor $\mathbf{S}$ for upper-convected Maxwell fluid obeys the following relation:

$$\left(1 + \lambda_1 \frac{D}{Dt}\right)\mathbf{S} = \mu \mathbf{A}_1, \tag{3}$$

where λ_1 is the fluid relaxation time, $\mathbf{A}_1 = (\nabla\mathbf{V}) + (\nabla\mathbf{V})^t$ the first Rivlin–Ericksen tensor and $\frac{D}{Dt}$ the upper-convected time derivative. For a second rank tensor $\mathbf{S}$ and a vector $\mathbf{a}$, we have the following:

$$\frac{D\mathbf{S}}{Dt} = \frac{\partial\mathbf{S}}{\partial t} + (\mathbf{V}\cdot\nabla)\mathbf{S} - \mathbf{LS} - \mathbf{SL}^t, \tag{4}$$

$$\frac{D\mathbf{a}}{Dt} = \frac{\partial\mathbf{a}}{\partial t} + (\mathbf{V}\cdot\nabla)\mathbf{a} - \mathbf{La}. \tag{5}$$

Now assigning the operator $(1 + \lambda_1 \frac{D}{Dt})$ to Eq. (2) and then using Eqs. (4) and (5), the component forms of the resulting equation after using boundary layer approximations are obtained as follows:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} - 2\Omega v + \lambda_1 \begin{bmatrix} u^2\frac{\partial^2 u}{\partial x^2} + v^2\frac{\partial^2 u}{\partial y^2} + w^2\frac{\partial^2 u}{\partial z^2} \\ +2uv\frac{\partial^2 u}{\partial x\partial y} + 2vw\frac{\partial^2 u}{\partial y\partial z} + 2uw\frac{\partial^2 u}{\partial x\partial z} \\ -2\Omega\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) + 2\Omega\left(v\frac{\partial u}{\partial x} - u\frac{\partial u}{\partial y}\right) \end{bmatrix} = \nu\left[\frac{\partial^2 u}{\partial z^2}\right], \tag{6}$$

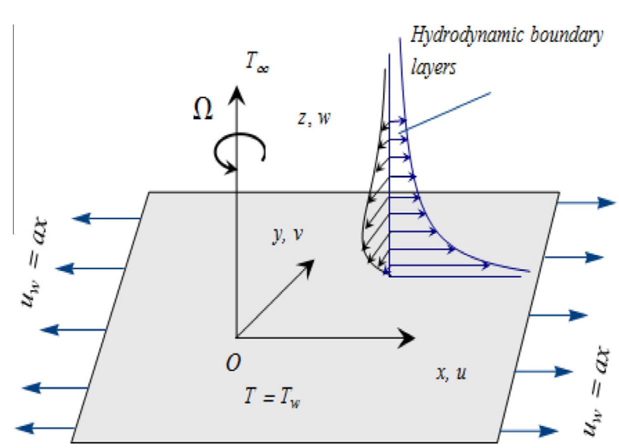

Fig. 1. Physical configuration and coordinate system.