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A modified spectral method for simulating arbitrary directional radiative intensity in participating media with graded refractive index

Lin-Yang Wei, Hong Qi *, Ya-Tao Ren, Li-Ming Ruan *

School of Energy Science and Engineering, Harbin Institute of Technology, Harbin 150001, PR China

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ABSTRACT

Accurate simulation of the arbitrary directional radiative intensity in participating media with graded refractive index (GRI) is important for solving the inverse radiation problem. The conventional spectral method (SM) can be applied to solve the radiative transfer problem of graded index media with limit-fixed discrete directions, which cannot obtain arbitrary directional radiative intensities required in inverse estimation. From the conventional SM, a modified spectral method (MSM) is proposed to simulate the arbitrary directional radiative intensity of participating media with GRI accurately. The detailed derivation process of MSM is illustrated and verified by solving the radiative transfer problem involving three different kinds of GRI distributions. All results show that the arbitrary directional radiative intensity can be calculated accurately based on the fixed directional radiative intensities of SM. Furthermore, the GRI distribution was reconstructed accurately by using the stochastic particle swarm optimization with noisy data and the MSM as forward method.

1. Introduction

Detecting the radiative signals among small solid angles or the radiative energy in a small zone of interest has attracted considerable attention because of its wide applications in various fields such as infrared target detection, optical tomography, combustion diagnosis, optical measurement of flame, medical diagnosis, measurement of thermophysical properties, and atmosphere remote sensing, to name a few [1–8]. Theoretically, the essence of all these measurement techniques is to solve the inverse problem, which means using the measurement signals to retrieve the physical parameters by solving the inverse problem. With this idea in mind, the directional radiative intensities can supply more information for these measurement techniques, which is important for solving the inverse problem effectively. Meanwhile, the increasing use and availability of high-resolution directional radiative sensors for many measurement applications requires simulating arbitrary directional radiative intensity accurately. Thus, obtaining the directional information of radiative transfer accurately and efficiently is imperative. Having a clear understanding and a reliable model of directional photon transport in participating media plays a significant role in improving techniques used for various measuring problems.

Radiative heat transfer (RHT) in a semitransparent medium with variable spatial refractive index (in other words, graded refractive index, GRI) has recently been of interest in thermo-optical systems and has evoked the interest of many researchers. In graded index media, the ray goes along a curved path determined by the Fermat principle. As a result, the solution of radiative transfer in graded index media is more difficult than that in uniform index media. Curved ray tracing is the main difficulty for the solution of radiative transfer in a medium with variable spatial refractive index. During the past three decades, various approximate and numerical methods have been developed to solve the radiative transfer problem of graded index media. The first pioneering treatment of radiation transport in semitransparent media with GRI was published by Siegel and Spuckler [5]. They indicated that the refractive indices of semitransparent sublayers inside a composite could have a considerable effect on the temperature distribution and radiative flux fields. Since then, the significance of GRI for the radiative properties and radiative transfer of semitransparent media has received increasing recognition and considerable attention. Numerous other attempts have been made to investigate the radiative transfer in participating media with GRI. For instance, Ben Abdallah and Le Dez et. al. [9–12] developed a curved ray tracing technique to analyze RHT in semitransparent media with GRI. From the works of Ben Abdallah and Le Dez,

* Corresponding authors at: School of Energy Science and Engineering, Harbin Institute of Technology, 92, West Dazhi Street, Harbin 150001, PR China.
E-mail addresses: qihong@hit.edu.cn (H. Qi), ruanlm@hit.edu.cn (L.-M. Ruan).

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Nomenclature

a	the coefficient of graded refractive index distribution $n(x)$
b	the coefficient of graded refractive index distribution $n(x)$
c	the coefficient of graded refractive index distribution $n(x)$
L	the thickness of medium
F	objective function
I	radiative intensity, W/(m^2 sr)
n	the graded refractive index of participating media
s	the curvilinear abscissa of the trajectory
S	the source term
M	the number of discrete directions
T	the temperature, K

Greeks symbols

χ	the derivative of refractive index with respect to the coordinate x
δ	the standard deviation
ε	the wall emissivity
Ω	direction of radiative intensity
Φ	the scattering phase function
γ	the measurement errors,%
κ_a	the absorption coefficient, m^{-1}
κ_s	the scattering coefficient, m^{-1}
μ, η, ξ	the direction cosine of the direction of light propagation
μ'	incoming direction cosine of radiative intensity
σ	Stefan–Boltzmann constant or the standard deviation
τ	optical thickness
ω	scattering albedo
ψ	the dimensionless radiative intensity
ς	random variable

Subscripts

0	the left boundary
L	the right boundary
b	blackbody
est	estimated value
ref	reference resources
i	ith note
mea	measured value
start	starting point

Huang et al. [13–16] presented a combined curved ray tracing and pseudo-source adding method for RHT in one-dimensional (1D) semitransparent medium with GRI. Liu [17] developed a discrete curved ray tracing method to analyze the radiative transfer in 1D absorbing–emitting semitransparent slab with variable spatial refractive index, in which the curved ray trajectory is locally considered a straight line, to overcome the complicated and time-consuming computation of ray trajectory. Consequently, Liu and Tan [18] adopted the discrete curved ray tracing method to analyze the transient temperature response in participating media with graded index under pulse irradiation. Lemonnier et al. [19] developed a discrete coordinate method to investigate RHT in 1D semitransparent media with GRI. Liu [20] developed a finite volume method to solve RHT in multi-dimensional GRI media. Xia et al. [21] adopted the combined Monte Carlo method and discrete curved ray tracing method to analyze the RHT problem in absorbing and scattering media with GRI.

In summary, various methods were employed to solve the radiative transfer problem in participating media with GRI. In contrast to the aforementioned methods, the spectral method (SM) is a high-order numerical method based on Lagrange interpolation polynomials and spectral collocation points [22]. In more recent years, the SM has attracted significant attention because of its wide potential applications in solving the radiative transfer equation (RTE). The SM [23] can provide exponential convergence and high accuracy with relative few grid points, and has advantages over other numerical methods in accuracy and efficiency. Moreover, the SM has been extended and applied to various heat transfer problems, including the thermal radiation heat transfer [24], and combined radiative–convective or radiative–conductive transfer [25–28], because of its high accuracy and mathematical simplicity.

Numerous studies on SM mainly focused on obtaining the temperature distribution in graded index media with fixed discrete directions. By contrast, few studies are concerned with the directional distribution of radiative intensities. From the experimental point of view, if the numerical method does not allow the simulation of sufficient radiative energy in arbitrary directions far from the normal to the surface of the sample, the acquired information will be insufficient to identify the radiative properties including GRI, absorption, scattering coefficient, and the phase function, to name a few. Thus, the aim of the present study is to simulate the directional radiative signals effectively. However, the existing techniques to solve the directional radiative intensity problem in participating media with GRI are rather limited. To the best of our knowledge, no research to date has used SM to solve the arbitrary directional radiative transfer in participating media with GRI and little attention has been focused on retrieving the GRI distribution efficiently from the measured directional radiative intensities.

From the previously presented discussion, the main objective of the present study is to develop a modified spectral method (MSM) to simulate the arbitrary directional radiative intensity of participating media with GRI. The integral form of radiative transfer equation is used to solve the directional radiative intensity by tracing the propagation path of the light in the medium. Ray goes along a curved path caused by variation of the refractive index. The MSM is utilized as forward solution to obtain the radiative intensity. In the numerical experiment, the results of radiative intensities obtained by MSM are used to reconstruct different kinds of GRI distribution in participating media. The remainder of this paper is organized as follows: The derivation of the RTE by MSM in participating media with GRI is introduced in Section 2. The computing method and simulation results of MSM are presented in Section 3. Three kinds of GRI distributions are reconstructed based on the directional radiative intensities by stochastic particle swarm optimization (SPSO) to evaluate the applicability of the MSM in Section 4. The main conclusions and perspectives are given in Section 5.

2. Theoretical model

Curved ray tracing is difficult and complex in a GRI medium because the light ray goes along a curved path determined by the Fermat principle. As a result, the solution of arbitrary directional intensity is more difficult than that in uniform index media. The MSM is developed to simulate the radiative intensity in any specified direction to overcome this difficulty. As shown in Fig. 1, a 1D participating medium with GRI is bounded by connected gray surfaces. The RTE can be written as [24]