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Modification of SIMPLE algorithm to handle natural convection flows with zero-isothermal compressibility

Yutaka Asako [a,*], Mohammad Faghri [b]

[a] Department of Mechanical Precision Engineering, Malaysia-Japan International Institute of Technology, University Technology Malaysia, Jalan Sultan Yahya Petra, 54100 Kuala Lumpur, Malaysia

[b] Department of Mechanical, Industrial and System Engineering, University of Rhode Island, Kingston, RI 02881-0805, USA

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ABSTRACT

The conventional SIMPLE algorithm for the pressure–velocity coupling has been adopted by many commercial and non-commercial CFD codes. It encounters convergence problem when it is used to solve unsteady natural convection flows with zero-isothermal compressibility. In this paper, a modified version of this algorithm is proposed to remedy this drawback. The modification includes updating of the density at each time step based on its value at the previous time step to satisfy the continuity equation. As an example of utilizing the modified SIMPLE algorithm, the unsteady natural convection in a rectangular cavity with isothermal vertical walls and adiabatic horizontal walls was computed. Physically consistent results were obtained.

1. Introduction

The SIMPLE algorithm [1] for the pressure–velocity coupling has been adopted by many commercial and non-commercial CFD codes such as FLUENT, Star-CD, Phoenix, OpenForm, etc. Both steady and unsteady natural convection in a cavity are solved by this algorithm by using the Boussinesq approximation. This approximation ignores the density changes except for the density that appears in the buoyancy term. It also assumes the density has a linear dependence on temperature. Therefore, Boussinesq approximation is not suitable for simulation of natural convection problems with large temperature changes [2,3] and also large density changes that exist in super critical fluids near pseudo-critical points. Recently, there has been renewed interest in applications which require precise solutions of natural convection in fluids with large variations in density. A numerical simulation using a non-Boussinesq approximation is required where the variable density in all the terms must be considered. There is no difficulty in taking into account the variations in density in the buoyancy term instead of using the assumption of the linear dependence on temperature. However, if the variable density is taken into account in all the terms, the SIMPLE algorithm encounters a convergence problem when it is used to solve unsteady forced convection problems. Matsushita [4] proposed a modification of the outflow boundary condition for an unsteady forced convection in a duct. The treatment of the outflow boundary is modified for the variable density. However, a careful search of the literature failed to disclose any prior work on a natural convection with variable density in a cavity.

This study deals with the modification of SIMPLE algorithm for natural convection for an idealized fluid in a cavity with non-zero isobaric compressibility (coefficient of thermal expansion), $-(\partial\rho/\partial T)_p/\rho$, but with zero-isothermal compressibility, $(\partial\rho/\partial p)_T/\rho$.

The ratio of isothermal to isobaric compressibility for an ideal gas can be expressed as T/p. This ratio is about 0.003 under atmospheric pressure and temperature conditions (10^5 Pa and 300 K). For water, this ratio is about 1.6×10^{-6} [5]. However, for super critical water it is about 3×10^{-6} and is about 6×10^{-6} for carbon dioxide near the pseudo-critical point [5]. Therefore, zero-isothermal compressibility can be assumed under these conditions.

2. An example problem using SIMPLE algorithm

To demonstrate the convergence problem of SIMPLE algorithm, consider an unsteady natural convection in a rectangular cavity with vertical isothermal walls and horizontal adiabatic walls as shown in Fig. 1. All the temperatures are maintained at T_C in the initial state. There is no driving force acting on the fluid and therefore the fluid is assumed to be initially stationary. At $t = 0$, the temperature of the left vertical wall (hot wall) is suddenly raised to T_H from T_C in a stepwise fashion.

* Corresponding author.
E-mail addresses: y.asako@utm.my (Y. Asako), faghrim@uri.edu (M. Faghri).

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Nomenclature

a	coefficient of discretization equation
b	source term of discretization equation
C_p	specific heat (W/(kg K))
Ga	Galilei number (–)
H	height of cavity (m)
L	width of cavity (m)
p	pressure (Pa)
P	dimensionless pressure (–)
Pr	Prandtl number (–)
R^C	normalized residual of continuity equation
R^U, R^V	normalized residual of U and V
S_{max}	maximum of rate of mass imbalance in each control volume
S_{sum}	rate of mass imbalance in cavity
t	time (s)
u, v	velocity components (m/s)
U, V	dimensionless velocity components (–)
V	volume integral (m^3)
x, y	coordinates (m)
X, Y	dimensionless coordinates (–)

Greek

β	volume expansivity (1/K)
ϕ	dependent variable
λ	thermal conductivity (W/(m K))
μ	viscosity (Pa s)
ν	kinetic viscosity (m^2/s)
θ	dimensionless temperature (–)
ρ	density (kg/m^3)
ρ^*	dimensionless density (–)
τ	dimensionless time (–)

Subscript

C	cold wall
e, w, n, s	control-volume faces
H	hot wall
nb	neighbor-point
P	central grid point
ref	reference

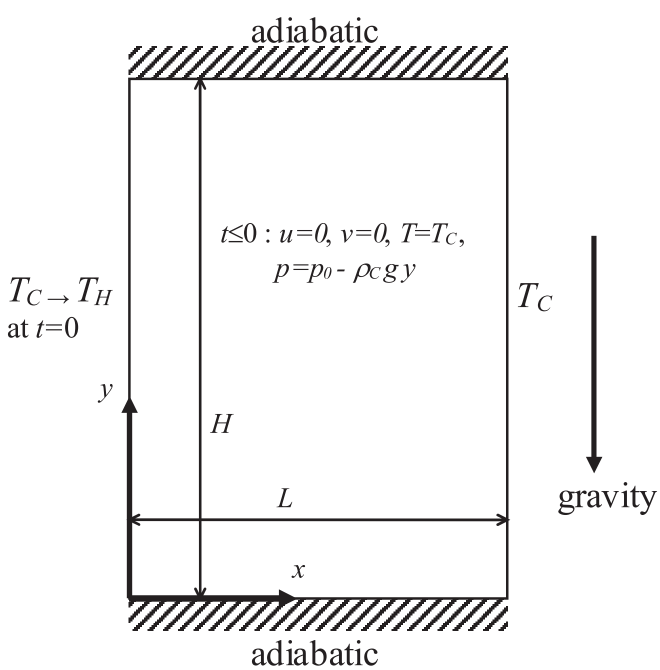

Fig. 1. Natural convection in a rectangular cavity.

2.1. Governing equations

The flow in the rectangular cavity is assumed to be two-dimensional. It is further assumed that all the fluid properties except the density are constant and also the fluid density is only a function of temperature as

$$\rho = \frac{const.}{T} \tag{1}$$

The density at a reference temperature, T_{ref}, is denoted by ρ_{ref} and the density ratio is expressed as $\rho/\rho_{ref} = T_{ref}/T$. The density ratio, ρ/ρ_{300} is plotted as a function of T for $T_{ref} = 300$ K in Fig. 2. The variations of the density ratio of water and super critical carbon dioxide at 8 MPa are also plotted in this figure as a reference. As can be seen in this figure, the density change of super critical carbon dioxide near the pseudo-critical point is large.

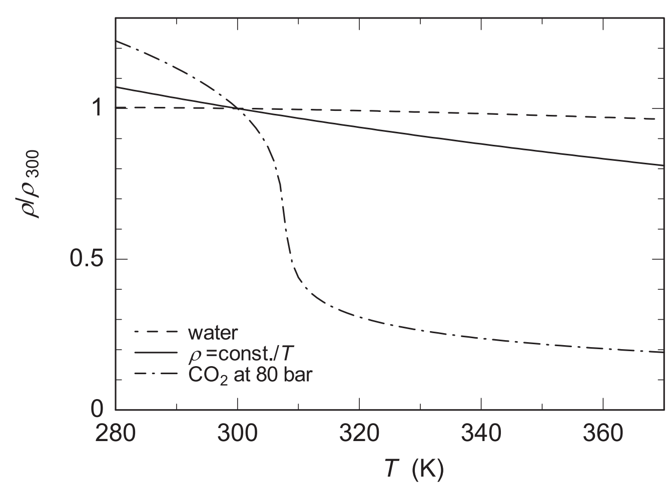

Fig. 2. ρ/ρ_{300} as a function of T.

Under these assumptions, the variable density needs to be taken into account for all the terms and the continuity and the momentum equations can be expressed as:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} = 0 \tag{2}$$

$$\frac{\partial \rho u}{\partial t} + \frac{\partial \rho u u}{\partial x} + \frac{\partial \rho u v}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\left(\mu \frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) + \frac{\mu}{3}\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) \tag{3}$$

$$\frac{\partial \rho v}{\partial t} + \frac{\partial \rho u v}{\partial x} + \frac{\partial \rho v v}{\partial y} = -\frac{\partial p}{\partial y} + \frac{\partial}{\partial x}\left(\mu \frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu \frac{\partial v}{\partial y}\right) + \frac{\mu}{3}\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) - \rho g \tag{4}$$