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Effectiveness of entropy generation and energy transfer on peristaltic flow of Jeffrey material with Darcy resistance

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ABSTRACT

The primary goal of present article is to analyze the entropy generation on peristaltic flow of Jeffrey material in a curved configuration. Velocity and thermal slip conditions are invoked. An incompressible fluid in a channel saturates the porous space. Modelling is based upon modified Darcy's law for Jeffrey fluid. Large wavelength and low Reynolds number approximations are utilized. Exact solutions of the resulting system of differential equations with corresponding boundary conditions are computed. Further analysis is made for the pressure gradient, stream function, velocity of the fluid, temperature, entropy generation and Bejan number. It is found that entropy generation and Bejan number are more visible in the vicinity of the channel walls when compared at the channel center.

1. Introduction

The engineers and scientists at present have great interest in the topic of entropy generation due to its extensive usage in heat exchangers, electronic cooling, turbo machinery, porous media, solar collectors, chemical vapor deposition instruments and combustions. Recently several scientific articles have been published for the analysis of physiological processes. However these physiological processes are very complicated systems that despite the fact that they display a specific kind of linearity, they likewise demonstrate disorderly and erratic conduct. Additionally in such sorts of systems included, the investigation of heart rate includes a progression of intriguing elements which emerge from an intricate blend of both deterministic and stochastic physiological procedures. Blood pressure oscillation also belongs to the category of such system while patients experience their normal routine work. Ambulatory blood pressure monitoring is a clinical process to estimate blood pressure every 30–60 min during 24 or 48 h. Also blood flow rises when a person performs hard physical activity and in such cases the blood circulation cannot stay normal. At the point when the surrounding temperature surpasses 20 °C, heat transfer carrying from the skin surface by the means of evaporation through sweating while under 20 °C human bodies loses heat through the process of conduction and radiation. To handle such type of serious situations entropy generation has a vital role to exactly identify such systems. The improvement of thermal systems has gained a notable consideration. Thermal systems have been explored by employing the second law of thermodynamics. According to second law of thermodynamics the accessible energy (Exergy) is always demolished mostly or absolutely and terminated quantity of energy is proportional to the entropy production. The execution of thermal gadgets is continuously transformed by irreversible dissipation that leads to an expansion of entropy and decline of thermal proficiency. Along these facts, the entropy generation decays or minimize the decimation of energy identification with the best productive energy framework plan. In engineering systems there are several references for entropy generation. Viscous dissipation, heat transfer, chemical reaction and electrical conduction are the basic sources of entropy generation in thermal systems. Bejan [1,2] explored the different constructive factors beyond the entropy generation in applied engineering, where spoliation of available work of a system exists throughout the entropy creation. Also entropy generation number $\left(N_s = \frac{S_{gen}}{S_g}\right)$ is suggested by Bejan. Very recently Bejan [3] reviewed the entropy generation minimization (or thermodynamic optimization) of flow configurations in engineering flow systems. Here design developments have been demonstrated by spreading the defects (e.g. stream resistances) throughout the system. Several researchers [4–10] have discussed the irreversibility shapes and entropy generation for different geometries.

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Peristaltic transport of liquids in a channel/tube has acquired a unique status among the recent researchers. Peristalsis is a wave like mechanism that happens because of involuntary contraction and expansion of elastic walls. There are several application of peristaltic mechanism have been developed in various fields of engineering and physiological systems. During bypass surgery it is utilized for the circulation of blood in heart lung machine. Naturally such kind of phenomenon appeared in the stomach, intestines and esophagus. Peristaltic phenomenon is the presentation of two critical reflexes that are animated by a bolus of nourishment stuff in the lumen. Peristaltic system has prompted the assembling of mechanical pumps wherein we can stay away from direct contact of the transported liquid with any moving parts, for example, plungers, rotors and valves. It also helped in the manufacturing of finger and roller pumping machines which are used as a part of pumping component during open heart surgery, infusion and dialysis pumps. In this direction, several studies on the peristaltic flow of Newtonian [11–14] and non-Newtonian [15–20] fluids have been presented since the pioneering analysis of Latham [21] and Shapiro et al. [22].

The porous medium consideration is quite significant in several geophysical processes. The common examples of porous medium are chunk of bread, wood, sandstone, limestone and shoreline sand. Gall bladder, human lungs and kidneys are the examples of porous medium in human body. The examination of blood flow through conduits are of extensive significance in numerous cardiovascular diseases especially atherosclerosis. In some obsessive circumstances, the dissemination of fatty cholesterol and corridor clogging blood clusters in the lumen of coronary vein can be considered equivalent to a porous medium. Few specialists have used the general Darcy's law in their analysis for porous medium. Experimental study of Beavers and Joseph [23] determines the flow of fluid at the boundary between the fluid layers and the porous medium. Thus suggested boundary conditions with slip at the interface. Hayat et al. [24] discussed the peristaltic motion of Maxwell with Hall effects in the presence of Darcy resistance. Some suitable studies [25–28] on the peristaltic flow with porous medium are important for the understanding.

It is examined that all the above mentioned analysis for peristalsis have been performed in straight channels which seems not realistic always because most of the arteries, glandular ducts and pipes are curved. Hence some developments have been made for peristalsis using curvilinear coordinates. Sato et al. [29] firstly established such investigation for peristaltic movement of viscous liquids. Ali et al. [30] extended the analysis of Sato et al. [29] using large wavelength approximation. Most recently some attempts [31–36] have been made for the impact of curvature on peristalsis of fluids in a channel. Mostly the heat transfer in peristalsis is examined either through imposed temperature or heat flux at the channel walls. Very little is yet explored for peristalsis via convective conditions at the channel walls (see [37–40]).

To our best information, the peristalsis of non-Newtonian fluid saturating porous medium in a curved channel is not studied so far. Here we aimed to study such flow for a Jeffrey fluid model. Analysis has been carried out when no-slip conditions for velocity and temperature do not hold. Entropy generation analysis is also addressed. The relevant problems are modeled and solved for various physical quantities of interest. Exact solutions are presented and analyzed.

2. Problem development

An incompressible Jeffrey material in curved channel (of thickness $2a_1$) twisted in a circle of radius R^* and center at O is considered. Peristalsis is due to waves propagation along the channel walls. We denote $\overline{V}_1$ and $\overline{V}_2$ the velocities along the axial ($\overline{X}$) and radius ($\overline{R}$) directions. Here fluid saturates the porous medium. Modified Darcy's law is employed. Entropy generation is also discussed through heat transfer and viscous dissipation. Physical model is shown in Fig. 1.

The waves have been written in the forms:

$$\overline{H}(\overline{X},\overline{t}) = a_1 + b_1 \sin\left(\frac{2\pi}{\lambda^*}(\overline{X} - c\overline{t})\right). \tag{1}$$

Here b_1 denotes amplitude, c wave speed, λ^* length of the wave and $\overline{t}$ the time in laboratory frame. The velocity $\overline{\mathbf{V}}$ is

$$\overline{\mathbf{V}} = (\overline{V}_1(\overline{X},\overline{R},\overline{t}), \overline{V}_2(\overline{X},\overline{R},\overline{t}), 0). \tag{2}$$

In Jeffrey fluid, the extra stress tensor satisfies [37,41]:

$$\overline{\mathbf{S}} = \frac{\mu}{1+\lambda_1}\left(\overline{\mathbf{A}}_1 + \lambda_2 \frac{d}{dt}\overline{\mathbf{A}}_1\right). \tag{3}$$

In above equation μ, λ_1, λ_2, $\frac{d}{dt}$ are the dynamic viscosity, ratio of relaxation to retardation times, retardation time and material time derivative. $\overline{\mathbf{A}}_1$ and $\frac{d}{dt}$ are

$$\overline{\mathbf{A}}_1 = (\text{grad}\,\overline{\mathbf{V}}) + (\mathbf{grad}\,\overline{\mathbf{V}})^{\mathbf{T}}, \tag{4}$$

$$\frac{d}{dt}\overline{\mathbf{A}}_1 = \frac{\partial}{\partial \overline{t}}(\overline{\mathbf{A}}_1) + (\overline{\mathbf{V}}.\nabla)\overline{\mathbf{A}}_1, \tag{5}$$

where T signifies matrix transpose. The basic equations which govern the flow are [30,32,36]:

$$\frac{\partial}{\partial \overline{R}}\{(R^*+\overline{R})V_1\} + R^*\frac{\partial \overline{V}_2}{\partial \overline{X}} = 0, \tag{6}$$

$$\begin{aligned}&\rho\left[\frac{\partial \overline{V}_1}{\partial \overline{t}} + \overline{V}_1\frac{\partial \overline{V}_1}{\partial \overline{R}} + \frac{R^*\overline{V}_2}{R^*+\overline{R}}\frac{\partial \overline{V}_1}{\partial \overline{X}} - \frac{\overline{V}_2^2}{R^*+\overline{R}}\right] = -\frac{\partial \overline{P}}{\partial \overline{R}}\\ &+\frac{1}{R^*+\overline{R}}\frac{\partial}{\partial \overline{R}}\{(R^*+\overline{R})\overline{S}_{\overline{R}\overline{R}}\} + \frac{R^*}{R^*+\overline{R}}\frac{\partial \overline{S}_{\overline{R}\overline{X}}}{\partial \overline{X}} - \frac{\overline{S}_{\overline{X}\overline{X}}}{R^*+\overline{R}} + R_{\overline{R}},\end{aligned} \tag{7}$$

$$\begin{aligned}&\rho\left[\frac{\partial \overline{V}_2}{\partial \overline{t}} + \overline{V}_1\frac{\partial \overline{V}_2}{\partial \overline{R}} + \frac{R^*\overline{V}_2}{R^*+\overline{R}}\frac{\partial \overline{V}_2}{\partial \overline{X}} - \frac{\overline{V}_1\overline{V}_2}{R^*+\overline{R}}\right] = -\frac{R^*}{R^*+\overline{R}}\frac{\partial \overline{P}}{\partial \overline{X}}\\ &+\frac{1}{(R^*+\overline{R})^2}\frac{\partial}{\partial \overline{R}}\{(R^*+\overline{R})^2\,\overline{S}_{\overline{R}\overline{X}}\} + \frac{R^*}{R^*+\overline{R}}\frac{\partial \overline{S}_{\overline{X}\overline{X}}}{\partial \overline{X}} + R_{\overline{X}},\end{aligned} \tag{8}$$

$$\begin{aligned}&\rho c_p\left[\frac{\partial T}{\partial \overline{t}} + \overline{V}_1\frac{\partial T}{\partial \overline{R}} + \frac{\overline{V}_2 R^*}{R^*+\overline{R}}\frac{\partial T}{\partial \overline{X}}\right] = \kappa\left[\frac{\partial^2 T}{\partial \overline{R}^2} + \frac{1}{R^*+\overline{R}}\frac{\partial T}{\partial \overline{R}} + \frac{R^{*2}}{(R^*+\overline{R})^2}\frac{\partial^2 T}{\partial \overline{X}^2}\right]\\ &+\mu\left[\left(\frac{\partial \overline{V}_2}{\partial \overline{R}} - \frac{\overline{V}_2}{(R^*+\overline{R})} + \frac{R^*}{(R^*+\overline{R})}\frac{\partial V_1}{\partial \overline{X}}\right)S_{\overline{R}\overline{X}} + \frac{\partial \overline{V}_2}{\partial \overline{R}}(S_{\overline{R}\overline{R}} - S_{\overline{X}\overline{X}})\right].\end{aligned} \tag{9}$$

In above equations $\overline{S}_{\overline{X}\overline{X}}, \overline{S}_{\overline{R}\overline{R}}, \overline{S}_{\overline{R}\overline{X}}$ are the extra stress components and $\overline{T}$ the fluid temperature in laboratory frame. Note that unsteady flow in fixed frame $(\overline{R},\overline{X})$ can be treated steady in wave frame $(\overline{r},\overline{x})$. The Darcy resistance in case of Jeffrey fluid model is

$$\mathbf{R} = -\frac{\mu}{k'(1+\lambda_1)}\left(1+\lambda_2\frac{d}{dt}\right)\overline{\mathbf{V}}. \tag{10}$$

The velocity and thermal slip conditions are imposed as follows:

$$\overline{V}_2 \pm \alpha_1\tilde{n}\frac{\partial \overline{S}_{\overline{R}\overline{X}}}{\partial \overline{R}} = \overline{U}_w,\ \overline{R} = \pm\overline{H}(\overline{X},\overline{t}), \tag{11}$$

$$(T - T_w) \pm \beta_1\tilde{n}\frac{\partial T}{\partial \overline{R}} = 0,\ \text{at}\ \overline{R} = \pm\overline{H}(\overline{X},\overline{t}), \tag{12}$$