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Thermally stratified stretching flow with Cattaneo–Christov heat flux

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ABSTRACT

The present article addresses the stagnation point flow of Jeffrey liquid towards a stretching cylinder. Heat transfer is analyzed in view of non-Fourier heat flux and thermal stratification. Expression of heat flux is based upon Cattaneo–Christov theory. Cattaneo–Christov heat flux model is utilized for the development of energy equation. Such consideration accounts the contribution by thermal relaxation. The series solutions for resulting flow and heat transfer problems have been computed. Interval of convergence for the obtained series solutions is explicitly determined. Physical quantities of interest have been examined for the influential variables entering into the problems. It is observed that velocity profile shows decreasing behavior for larger Deborah number. Further that temperature distribution decreases for larger values of thermally stratification and thermal relaxation parameters.

1. Introduction

Heat transfer mostly in the past has been addressed using classical Fourier's law of heat conduction [1]. Energy equation via Fourier's law is parabolic. It shows that the whole system is instantly affected by the initial disturbance. This issue has been controlled through the thermal relaxation time in the Fourier's law (see Cattaneo [2]). Energy equation subject to Cattaneo–Christov heat flux yields hyperbolic partial differential equation [3,4]. Christov [5] improved the analysis of Cattaneo [2] by introducing thermal relaxation time and using Oldroyd's upper-convected derivatives for the material-invariant formulation. Han et al. [6] studied Cattaneo–Christov heat flux in the stretched flow of Maxwell fluid over a surface with constant thickness. Thermal conductivity of liquid is assumed constant. Straughan [7] utilized Cattaneo–Christov model for thermal convection in an incompressible flow of viscous fluid. Structural stability and uniqueness of the Cattaneo–Christov equations are also discussed by Ciarletta and Straughan [8]. Hayat et al. [9] employed Cattaneo–Christov heat flux in MHD flow of an Oldroyd-B fluid over a stretching surface with homogeneous/heterogenous reactions. Hayat et al. [10] also explored the impact of Cattaneo–Christov heat flux in the stretched flow over a variable thick surface. Mustafa et al. [11] analyzed rotating flow of magnetite-water nanofluid by a stretched sheet inspired by non-linear thermal radiation. Waqas et al. [12] studied Burgers fluid flow with Cattaneo–Christov heat flux in the presence of variable thermal conductivity. Three dimensional flow of Maxwell fluid with Cattaneo–Christov heat flux model is analyzed by Abbasi and Shehzad [13]. Hayat et al. [14] investigated Jeffrey fluid flow with Cattaneo–Christov heat flux due to variable thicked surface. Li et al. [15] analyzed heat transfer in MHD viscoelastic flow with Cattaneo–Christov heat flux model. Hayat et al. [16] examined stagnation point flow of Maxwell fluid with Cattaneo–Christov heat flux and homogeneous-heterogeneous reaction. Anomalous convection diffusion with Cattaneo–Christov heat flux due to coupling transport of cells is examined by Liu et al. [17]. Reddy et al. [18] considered three different geometries to analyze Cattaneo–Christov heat flux in presence of cross diffusion effects. Hayat et al. [19] examined two-dimensional stratified flow of Eyring-Powell fluid with Cattaneo–Christov heat flux. Impact of Cattaneo–Christov heat flux is addressed by Tanveer et al. [20]. Nadeem and Muhammad [21] reported stratified flow of Maxwell fluid utilizing Cattaneo–Christov heat flux theory.

The flow of nonlinear fluids has gained significant importance owing to its several applications in the fields of applied science and engineering. The traditional Navier–Stokes equation is not adequate to predict the characteristics of such flows. In general

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Nomenclature

u, v	velocity components	β	Deborah number
α	curvature parameter	A	ratio parameter
u_e	free stream velocity	c_p	specific heat
λ_1	ratio of relaxation to retardation	T	temperature
λ_2	retardation time	q	heat flux
λ_3	thermal relaxation time	k	thermal conductivity
U_0	stretching velocity	α^*	thermal diffusivity
ρ	density	l	characteristics length
υ	kinematic viscosity	c, d	dimensional constant
T_w, T_∞	fluid and ambient temperature	Pr	Prandtl number
r	radius of cylinder	τ_w	wall shear stress
C_f	skin friction coefficient	γ	thermal relaxation parameter
Re_x	Reynold number		
S	thermal stratification		

mathematical formulation for these fluids is complicated. Such liquids cannot be described through linear relationship between rate of strain and shear stress. The non-linear fluids are encountered in the chemical and petroleum procedures and geophysics. Materials for example boring muds, fruit purée, foams, soaps, sugar arrangement glues, certain oils, ketchup, lubricants, dirt covering, suspension arrangements and colloidal behave like the non-Newtonian fluids. Such fluids can be into the differential, integral and rate types. Relaxation and retardation times can be explained only through rate type materials. Jeffrey fluid is one of the subclasses of rate type materials explaining the relaxation and retardation times effects. Having this in mind, Hayat et al. [22] studied flow of Jeffrey fluid in the presence of radiation, heat source and porous medium. Hussain et al. [23] studied radiative hydromagnetic flow of Jeffrey nanofluid by an exponentially stretched surface. Farooq et al. [24] considered MHD flow of Jeffrey liquid in presence of Newtonian heating. Hayat et al. [25] described MHD stagnation point flow of Jeffrey nanofluid by taking into account Newtonian heating. Hamad et al. [26] investigated boundary layer stagnation point flow of variable thermal conductivity Jeffrey fluid over a stretching/shrinking surface. Hayat et al. [27] considered flow of Jeffrey fluid subject to Cattaneo–Christov heat flux and homogeneous/heterogeneous reaction. Tripathi et al. [28] discussed the MHD peristaltic flow of Jeffrey liquid in a cylindrical tube of finite length. Yasmeen et al. [29] investigated ferrofluid flow by a stretched surface in the presence of magnetic dipole and homogeneous-heterogeneous reactions. Reddy et al. [30] examined flow of Jeffrey fluid between torsionally oscillating disks.

Flow characteristics in the neighborhood of stagnation point is still a topic of hot interest for the recent scientists and researchers. Such interest mainly is due to its prominent demands in the industrial and engineering process. Stagnation point has significant role in flow of ground water since several streamlines advancing through them portray diverse flow regions. Hiemenz [31] initially reported the steady flow in the vicinity of stagnation point. Hayat et al. [32] explored convective flow of Maxwell liquid in the presence of thermal radiation and mixed convection. Stagnation point flow of hydromagnetic viscous liquid in presence of slip condition and homogeneous/heterogeneous reaction is studied by Abbas et al. [33]. Malvandi et al. [34] explored partial slip effect in the time-dependent stagnation point flow of viscous nanofluid. Impact of convective heat transfer in MHD flow of Jeffrey liquid towards a stretched surface is reported by Hayat et al. [35]. Joule heating effect in stagnation point flow by a cylinder is analyzed by Nawaz et al. [36].

Here we consider the impact of Cattaneo–Christov heat flux model in stagnation point flow of Jeffrey fluid. The flow is caused by a stretching cylinder. Further effect of thermal stratification is studied. Homptopic procedure [37–50] is utilized for the solutions of arising nonlinear differential systems. Results are discussed graphically for different sundry variables.

2. Formulation

2.1. Flow equations

Consider the stagnation point flow of non-Newtonian liquid towards a horizontal stretched cylinder. The cylinder is stretched with velocity $U_w(x) = U_0(x/l)$. We consider here the following assumptions.

(1) Two-dimensional flow:
(2) Jeffrey fluid model.
(3) Incompressible fluid.
(4) Thermal stratification.
(5) Cattaneo–Christov heat flux consideration.

The governing flow equations are

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial r} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} = U_e(x)\frac{dU_e(x)}{dx} + \frac{\mu}{\rho(1+\lambda_1)}\left(\frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial r^2} + \lambda_2\left(\begin{array}{c} \frac{v}{r}\frac{\partial^2 u}{\partial r^2} + \frac{\partial v}{\partial r}\frac{\partial^2 u}{\partial r^2} + v\frac{\partial^3 u}{\partial r^3} \\ +\frac{u}{r}\frac{\partial^2 u}{\partial x \partial r} + \frac{\partial u}{\partial r}\frac{\partial^2 u}{\partial x \partial r} + u\frac{\partial^3 u}{\partial x \partial r^2} \end{array}\right)\right), \tag{2}$$

subject to the appropriate boundary conditions

$$u = U_w(x) = \frac{U_0 x}{l},\ v = 0,\ \text{at } r = a, \tag{3}$$

$$u \rightarrow U_e(x) = \frac{U_\infty x}{l} \text{ when } r \rightarrow \infty. \tag{4}$$

In the above-mentioned expressions u and v are the velocity components parallel to the x and r directions respectively, ρ the density of fluid, ν the kinematic viscosity, $U_e(x)$ the free stream velocity, a the radius of cylinder and l the characteristics length. Using transformations

$$\eta = \sqrt{\frac{U_0}{\nu l}}\left(\frac{r^2 - a^2}{2a}\right), \quad u = \frac{U_0 x}{l}f'(\eta), \quad v = -\frac{a}{r}\sqrt{\frac{U_0 \nu}{l}}f(\eta), \tag{5}$$