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Statistical radiative modeling of a porous medium with semi transparent and transparent phases: Application to a felt of overlapping fibres

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ABSTRACT

A general statistical model of characterisation of the radiative properties of homogenised phases has been developed for a porous medium with a semi transparent absorbing phase a and a transparent one b, characterised by general interfacial reflection and transmission laws. For non Beerian homogenised phases, it is based on successive sets of radiative statistical functions: extinction cumulative distribution functions, scattering cumulative probabilities and general phase functions ab initio determined by a Monte Carlo approach, only from morphological data and interfacial reflection and transmission laws in the last case. Specific sets are associated with isotropic and uniform volume emission by a and with the successive internal and external scattering events within a and b, the emission or scattering source terms of which have been weighted by spatial distribution functions. For a Beerian homogenised phase, a unique set of radiative statistical functions has been determined from random isotropic volume source points.

Two Generalised Radiative Transfer Equations (GRTEs), coupled by external scattering source terms are then expressed only vs the radiative statistical functions. It is shown that a radiative Fourier's model, based on radiative conductivity tensors, is not valid for a medium made of a semi transparent phase and a transparent one, if the particular case for which the semi transparent phase becomes opaque and the trivi-al case of a quasi isothermal medium are excepted.

The previous models are applied to a felt of fibres for insulation of high temperature systems. The radiative power field in radiation steady state within a felt of fibres enclosed between parallel opaque walls has been determined by solving the coupled GRTEs by a Monte Carlo method, for different values of the transverse optical thickness of a fibre. The temperature field within the felt has also been determined for two temperatures imposed at the boundaries and for imposed flux and temperature. Finally the optimal conditions of insulation have been determined for a case such that usual conduction can be neglected compared to radiation.

1. Introduction

An accurate modeling of radiative transfer is required in applications involving heat transfer in a porous medium at high temperature. The radiative properties of the different phases have to be characterised separately in order to be able to account for coupling with other heat transfer modes, generally conduction in a solid phase and conduction or convection in a fluid phase. As radiative transfer cannot be in practice determined at local scale, effective properties of every phase have to be determined.

The methods of parameter identification with many variants based on experiments or numerical transfer simulation (see a detailed review in Ref. [1]) are the most popular methods of characterisation of these radiative effective properties. If they are relevant for media characterised by an exponential extinction law (Beerian media), their limitations for non Beerian media have been discussed elsewhere [2]. Moreover, these methods do not easily allow every phase to be separately characterised.

In the general case, the radiative effective properties of a phase of a porous medium, which is often statistically non homogeneous and anisotropic, are non Beerian. The conditions of validity of the Beer's law for characterising these radiative effective properties have been discussed in a recent paper [3]. The law is, in particular, valid if one of the following conditions is fulfilled:

(i) When the phase is statistically homogeneous at all the spatial scales considered in the application; It can be non homogeneous at scales such that the medium is optically thin;

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Nomenclature

Lattin symbols

n	normal unit vector towards the (semi) transparent phase
r	coordinates of a current point
u	unit vector of the current direction
q	flux ($\mathrm{W \cdot m^{-2}}$)
B	generalised extinction coefficient
f	distribution function
G	radiative cumulative distribution function
I	radiative intensity
k^R	radiative conductivity
M	volume source point
M'	extinction point
N	interfacial source point
n	refractive index
P	cumulative probability
p	phase function
S	radiative source term
s, s'	curvilinear abscissas along a ray
T	temperature
V	volume
z	axis of the bed f fibers
D	medium length
d	cylinder diameter
L	cylinder length

Greek symbols

β	extinction coefficient
κ	absorption coefficient
ν	frequency
ρ''	bidirectional reflectivity
τ''	bidirectional transmissivity
δ	Kronecker symbol
Ω	solid angle
Φ	Azimutal angle
Π	volume fraction or porosity
Σ	interfacial area
Σ_{cc} or Σ_{cd}	generalised scattering coefficient
θ	Angle related to the z axis

Subscripts

a	absorption
c	phase a or b
d	another phase a or b
ext	extinction
e	emission
H	successive set of internal and external scattering events
sc	scattering

Superscripts

(n)	nth set of scattering events
$\circ$	at equilibrium
B	related to a Beerian phase
S	related to an interfacial source

Others

GRTE	Generalised Radiative Transfer Equation
RTE	Radiative Transfer Equation
STT	Medium with Semi Transparent and Transparent phases

(ii) When the interfaces of the porous medium present some special symmetries: For instance outside of overlapping spheres or cylinders;

(iii) At spatial scales larger than δ, length such that the medium is optically thick. In this popular case, extinction, scattering and absorption coefficients can be defined and a radiative conductivity tensor can be introduced (radiative Fourier's law).

A radiative model which can be applied to non Beerian media has been initiated by Consalvi et al. [4] and developed by Lipinski et al. [5]. This approach is similar to the volume averaging method of Whitaker and Quintard [6,7], for obtaining effective properties of a porous medium in the case of small perturbations of the field within a representative volume element.

The development of X and γ tomography techniques has led in the last decades to an accurate knowledge of the morphology of porous media. On the other hand the morphology of material models, sets of overlapping or non overlapping spheres or cylinders for instance, is exactly known. In these conditions, the increasing power of the statistical Monte Carlo methods has allowed the radiative properties of a non Beerian homogenised phase to be exhaustively characterised by radiative statistical functions instead of extinction, absorption and scattering coefficients, valid for a Beerian medium. The principles of the method have been defined by Tancrez and Taine [8], for models of foams. A non Beerian homogenised phase is characterised by an extinction cumulative distribution function, a scattering (or absorption) cumulative probability and a general scattering phase function a priori depending on both the incidence and scattering directions. In this first work and in following ones, for instance in Refs. [9–15], the Beerian assumption has been validated for statistically isotropic and homogeneous media.

An original transfer model for a non Beerian homogenised phase, directly based on a Generalised Radiative Transfer Equation (GRTE) involving the radiative statistical functions, has then been developed [16] for a medium with an opaque and a transparent phase, both in the general case and when a radiative Fourier's law is valid. The physical bases of the GRTE have been improved in a recent paper [3]. This model has been applied, under the Fourier's assumption, to ordered sets of intact or degraded diffuse opaque parallel cylinders within a transparent or a semi transparent medium [14]. This porous medium, modeling a nuclear core, is strongly anisotropic but homogeneous at large scale. In a recent work [17], the GRTE has been applied to a statistically strongly non homogeneous porous medium: A set of non overlapping diffuse opaque spheres at the vicinity of a wall within a transparent medium.

A key feature of a non Beerian phase is that the knowledge of the incident intensity field at a given point M is not sufficient for allowing extinction by absorption, external scattering or internal scattering at this point M to be determined [3,16]: The history of all radiations issued from all source points which contribute to the intensity field at M has to be accounted for (memory effect). This phenomenon makes the model much more complex, in particular within a medium with transparent and semi transparent phases characterised by general (non diffuse) interfacial reflection and transmission laws: It is then necessary to account for all the previous paths involving different sets of effective internal and external scattering, associated with interfacial reflection and

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