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Three-dimensional inverse heat conduction modeling of a multi-layered hollow cylindrical tube using input estimation algorithm and thermal resistance network

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ABSTRACT

High-temperature gas flow can cause melt, crack, erosion, and wear of tubes. In order to reduce these side effects, the inner wall of a hollow cylindrical tube is generally coated with a resistive material, for example, a steel pipe coated with a chrome layer. In addition, it is important to know heat flux applied to the tube wall and temperature distribution in the tube wall for appropriate design of the hollow cylindrical tube. In this study, three-dimensional inverse heat conduction modeling of a multi-layered hollow cylindrical tube was conducted. The thermal resistance network (TRN) scheme was employed to solve the heat conduction in the tube. The temperature distribution in the tube was estimated from a measured temperature on the outer wall of the tube by Kalman filter. At the same time, unknown heat flux on the inner wall of the tube was calculated by the recursive least squares algorithm.

1. Introduction

Recently various analytical methods using numerical analysis have been introduced in various fields of engineering, and studies using these methods are in progress. These achievements resulted from not only the numerical analysis algorithm portraying physical phenomena but also the improvement of process for calculation. In order to predict a physical phenomenon of interest through the numerical analysis, interested region's physical property, boundary condition, and initial condition should be given besides the accurate modeling for the phenomenon. However, different methods of approach should be used instead of the ordinary analysis method if it is difficult to know the boundary conditions of the region being analyzed accurately due to difficulty of the measurement. For example, the direct heat transfer analysis, which is generally used in the field of heat transfer, makes prediction of internal temperature distribution possible because its mathematical modeling as well as boundary and initial conditions for the analyzed region are clear. When a certain boundary condition is unknown, the inverse heat transfer analysis is used to predict the boundary condition and the internal temperature distribution. For the inverse heat transfer analysis, mathematical modeling for the interested region is mandatory, and at the same time, mathematical tools are needed in order to draw physical quantity. Since the inverse heat transfer analysis is mathematically more complicated and requires a lot more numerical costs than the general direct heat transfer analysis, selection of numerical method is important to solve problems more effectively.

Studies on analytical or numerical methods have been done by many researchers for a long time to effectively solve the inverse heat transfer problem. First, as an analytical method, a method for predicting heat flux using heat conduction equation and temperature of two points was introduced. However, it could only analyze steady state condition and linear temperature distribution since it predicted heat flux by integrating heat conduction equation. To improve the weakness, different problem solving method using integral or Laplace transform technique was introduced by many researchers including Stolz [1–5]. However, it also had a limit that it could only analyze one-dimensional unsteady state condition of relatively simple form. As a numerical method, the sequential estimation method has been studied by researchers like Back et al. [6–9], and study of the conjugated gradient method is still in progress by research teams including Alifanov et al. [10–14]. Both the sequential estimation method and the conjugated gradient method are types of recursive calculation method, which is simple and effective, and are most widely used to solve both linear and nonlinear problems. In addition, the recursive

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Nomenclature

A	area, m^2
B	sensitivity matrix
C	capacitance, J/K
$[C]$	capacitance matrix
C_p	specific heat, J/kg·K
E	element number
$\{F\}$	thermal load vector
h	convection heat transfer coefficient, $W/m^2 \cdot K$
H	measurement matrix
I	identity matrix
k	thermal conductivity, W/m·K
K	Kalman gain
K_b	steady-state correction gain
L	length, m
M	sensitivity matrix
$[M]$	conductance matrix
O	error order
P	filter's error covariance matrix
P_b	error covariance matrix
q	heat flux, W/m^2
$\hat{q}$	estimated input vector, W/m^2
Q	process noise covariance
r	circumferential direction
R	measurement noise covariance
Re	thermal resistance, K/W
s	innovation covariance
t	time, s
T	temperature, K
$\{T\}$	temperature vector
T_∞	ambient temperature, K
T_0	initial temperature, K
V	volume, m^3
X	state vector
$\bar{X}$	input estimator
z	length direction
Z	observation vector
$\bar{Z}$	bias innovation

Greek letters

α	thermal diffusivity, m^2/s
γ	forgetting factor
Γ	input matrix
θ	time-stepping scheme parameter
κ	time (discretized), s
Λ	coefficient matrix
v	measurement noise vector
ρ	density, kg/m^3
ϕ	direction of angle
Φ	state transition matrix
ω	process noise vector

Superscripts

n	time domain

Subscripts

c	chrome
$cond$	conduction
$conv$	convection
f	interface between chrome and steel
H	heat flux
i	inner
l	point number of angle of direction
m	point number of length direction
n	point number of circumferential direction
o	outer
s	steel
$solar$	solar radiation energy
t	tube

input estimation algorithm, *i.e.*, a method for predicting unsteady state condition, provides information of changes in variables on time and draws result that is the closest to actual value of physical quantity by adopting the differences of each variable [15–18].

In this study, analysis of inverse heat transfer through a hollow cylindrical tube is performed with use of the recursive input estimation algorithm. For this study to be performed, numerical heat transfer modeling of the hollow cylindrical tube is needed while, in other existing studies, finite element method was used for system modeling [19–22]. A shortcoming of the finite element method, an existing model, is that numerical costs are very high since it uses so many high-dimensional numerical calculations. In this study, the thermal resistance network method, a type of the finite differential method, is used for heat transfer modeling in order to complement the shortcoming of the finite element method. Some advantages of using the thermal resistance network method are that it has a wide range of applications since it represents the derivative terms, which appear in differential equations, by using Taylor series expansion, and that it has relatively small quantity of numerical calculation. Due to these advantages, the thermal resistance method has been commonly used for the analysis of heat transfer that is used to predict the temperature distribution of steady or unsteady state condition within a motor [23–24].

In this study, development of an analysis model for prediction of heat flux and temperature distribution in the three-dimensional hollow cylindrical tube is performed with use of the inverse heat transfer analysis employing the Kalman filter and the recursive input estimation algorithm. During the process, the thermal resistance method is used for modeling in order to save numerical costs from the existing heat transfer modeling method. The way that the thermal resistance method is applied to the recursive input estimation algorithm is introduced. The inverse heat transfer analysis method that is developed in this study predicts heat flux, which variously changes according to changes in time and position, and proves its validity by comparing its result to the true value. Also, its prediction value of heat flux is compared to the true value when disturbance, such as solar radiation, happens outside the tube in order to evaluate its performance.

2. Model description

In this study, a simple tube as shown in Fig. 1 is used to develop a modeling for the inverse heat transfer analysis of the three-dimensional hollow cylindrical tube. The tube consists of two layers, chrome and steel, for its durability, since fluid with high temperature and pressure is assumed to flow in the tube. The tube's internal diameter, external diameter, and length are denoted as r_i, r_o, and L, respectively, and the interface between the boundaries of chrome and steel layers is denoted as r_f. Due to the flow of fluid with high temperature and pressure in the tube, heat flux, $q(z, \phi, t)$, which is a function of time, the tube's length, and direction of angle, can be calculated. However, it is assumed that the heat flux

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