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Three-dimensional numerical simulations of free convection in a layered porous enclosure

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ABSTRACT

Three-dimensional numerical simulations are carried out for the study of free convection in a layered porous enclosure heated from below and cooled from the top. The system is defined as a cubic porous enclosure comprising three layers, of which the external ones share constant physical properties and the internal layer is allowed to vary in both permeability and thermal conductivity. The model is based on Darcy's law and the Boussinesq approximation. A parametric study to evaluate the sensitivity of the Nusselt number to a decrease in the permeability of the internal layer shows that strong permeability contrasts are required to observe an appreciable drop in the Nusselt number. If additionally the thickness of the internal layer is increased, a further decrease in the Nusselt number is observed as long as the convective modes remain the same, if the convective modes change the Nusselt number may increase. Decreasing the thermal conductivity of the middle layer causes first an increment in the Nusselt number and then a drop. On the other hand, the Nusselt number decreases in an approximately linear trend when the thermal conductivity of the layer is increased.

1. Introduction

The problem of free convection in layered porous media has been of great interest in research due to the its presence in both nature and engineering processes. Geothermal reservoir and ground water modeling are examples of the application fields of this topic. Thermal gradients in fractured-porous media can drive convective flow [1], and create thermal anomalies of interest in geothermal applications [2–4]. The study of convective heat transfer in layered porous media is particularly important, since the presence of high (or low) permeability strata is a geological feature commonly found in hydrothermal systems. In this paper we present 3D steady-state numerical simulations of free convection in a three-layer porous enclosure.

Early work on the onset of convection in layered porous media is that by McKibbin and O'Sullivan [5,6]. They studied two and three-layer systems considering constant thermal conductivity in a two-dimensional cell. They defined a Rayleigh number referred to the physical properties of the bottom layer and the total thickness and temperature drop of the enclosure. From linear stability analysis they calculated critical values (Ra_c) as a function of the permeability ratio. They found that considerably high permeability ratios between layers ($\sim$20) are required to observe convective modes different from those for a homogeneous porous medium, these convective modes are characterized by some degree of confinement of convection in the high-permeability layers. Richard and Gounot [7] studied the onset of convection in a layered porous medium considering both anisotropic and isotropic layers as regards the permeability and thermal conductivity. As a particular case study, they calculated numerically Ra_c for the onset of convection for a two-layer porous medium with isotropic layers and showed that the stability of the system increases when the permeability of the upper layer is decreased, their definition of Ra was based on a weighted average of permeability and thermal conductivity on the thickness of the layers. The magnitude of this increase was in turn dependent on the relative thickness of the layers. In a similar two-layer model Rosenberg and Spera [8] reported an asymptotical increase in the Nusselt number as the permeability ratio of the top to the bottom layers was increased, they observed confinement of convection for a permeability ratio of the top to the bottom layers of 10 and $Ra = 35$ which was defined with respect to the bottom layer of the system. Mckibbin and Tyvand [9] investigated the conditions under which thermal convection in a layered porous medium can be comparable to that for an anisotropic porous medium. They pointed out that a multilayer system can be

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Nomenclature

β	thermal expansion coefficient (°C^{-1})	k	permeability (m^2)
$\boldsymbol{\psi}$	dimensionless vector potential	L	characteristic length (m)
η	thermal diffusivity (m^2/s)	Nu	Nusselt number
$\gamma(z)$	dimensionless smooth function	P	dimensionless pressure
μ	viscosity (Pa s)	Ra	Darcy-Rayleigh number
$\phi(z)$	dimensionless smooth function	Ra_c	critical Rayleigh number
ρ_0	density of reference (kg/m^3)	T	temperature (K)
θ	dimensionless temperature	t	dimensionless time
$\mathbf{e}$	unit vector in the z-coordinate	x, y, z	dimensionless coordinates
$\mathbf{u}$	dimensionless velocity		
g	gravitational constant (m/s^2)		

modeled by an analog anisotropic system when there is no confinement of convection in the layered system.

The problem of porous layers separated by conductive impermeable interfaces has also been investigated. Jang and Tsai [10] studied the onset of convection in a two-layer system separated by a conductive interface. They defined an overall Rayleigh number considering the total thickness of the arrangement of layers and found that the presence of the impermeable layer increases considerably the stability of the system, being the most stable those cases with the impermeable layer located in the middle of porous cell. More recently Rees and Genç [11] studied multilayer systems separated impermeable interfaces of negligible thickness and observed that Ra_c, defined locally in each layer, tends asymptotically to 12 as the number of sublayers is increased. Patil and Rees [12] extended the study to consider finite thickness of the conductive interfaces so that the conductivity had an impact on the behavior of the system. They reported that Ra_c and the associated wave number decreased when the thermal conductivity of the solid interfaces was decreased. Hewitt et al. [13] determined statistical steady-state convection at high Ra in a 2D periodic porous enclosure. Their model consists of a thin low permeability layer sandwiched by two high permeability layers. Regarding the convective modes, they found that for a given Ra and permeability ratio, an increase in the thickness leads to an ordered array of cells with stratification of the flow. On the other hand, they noted that the Nusselt number as a function of thickness of the low permeability layer experiences first a small increase for small thickness and then it decreases for larger thickness. Other free convection models of interest are those by Sheremet et al. [14]. These authors analyzed the steady-state convection of heat by taking into account Brownian diffusion and thermophoresis effects in nanofluids. Analogous analyses for heat generating porous enclosures in two-dimensional domains were reported by Grosan et al. [15] and Lam and Prakash [16]. Most recently, Zhu et al. [17] conducted a three-dimensional numerical study on the problem of natural convection in an anisotropic porous cube filled with a non-Newtonian fluid. They investigated the behaviors of Nusselt and Sherwood numbers and specified conduction and convection dominated regimes as well as the influential parameters. Importantly, none of these studies were primarily concerned with the multi-dimensional flow structures and their influences upon heat transfer performance of the system.

Although the scope of this work is layered porous media, it is important to mention the work by Nield and Kustnetzov [18,19] who investigated the effect of weak and moderate vertical and horizontal heterogeneities. They defined a Rayleigh number based on the mean properties of the porous enclosure and found that these heterogeneities lead to a decrease in Ra_c for all combinations of horizontal and vertical heterogeneities and all combinations of permeability and conductivity heterogeneities. Vertical heterogeneity proved to have greater influence than horizontal heterogeneity, presumably due to the influence of gravity. Likewise, Capone [20] found that an increase in the permeability in the upward direction is destabilizing whereas an increase in the downward direction is stabilizing. Nield and Kuznetsov [21] reported that horizontal variations in both permeability and thermal diffusivity lead to slight destabilization in comparison with vertical variations.

The aim of this study is to obtain 3D steady-state numerical solutions of free convection in a three-layer porous enclosure. The steady-state solutions are obtained from the simulation of the transient problem applying a convergence criterion. A parametric study is carried out to evaluate the Nusselt number as a function of the permeability, thermal conductivity, and thickness of the internal layer of the system.

2. Problem formulation

The porous enclosure consists of a three-layer system, of which the external layers have the same and constant physical properties and the internal may differ as regards the permeability and thermal conductivity (Fig. 1). It is assumed that the porous medium is isotropic within each layer. Fluid flow is governed by Darcy's law and buoyancy effects are described by the Boussinesq approximation. Local thermal equilibrium and negligible viscous heat generation are additional assumptions in this problem. From these considerations the momentum equation can be written in the following form (we use bar notation to denote dimensional variables and operators):

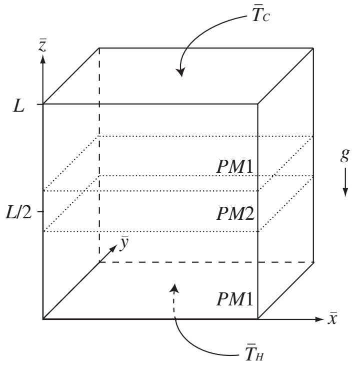

Fig. 1. Schematic model of a layered porous enclosure heated from below and cooled from the top with adiabatic lateral boundaries. The external layers ($PM1$) have constant properties, whereas the properties of $PM2$ are allowed to vary.

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