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An inverse method for the estimation of a long-duration surface heat flux on a finite solid

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ABSTRACT

The nonlinear inverse heat conduction problem (IHCP) is resolved for the estimation of surface heat flux from two temperature measurements inside a finite domain. An improvement is achieved in this study by optimizing the rescaled time used to linearize the heat conduction equation with temperature-dependent thermal properties. A closed-form solution of the transient temperature is derived, considering the inhomogeneous boundary conditions. The direct solution is further incorporated into a sequential technique for inverse analysis, and the uncertainty of inverse solution with respect to input parameters is obtained. The results from a representative example show that the method is computationally efficient and the inverse solution can be stabilized by increasing the rescaled time step size. Thus, the effect of the real time step on the ill-posedness can be reduced and the increase of sampling rate in experiment is possible. The present method is useful for the prediction of long-duration heat flux in thermal engineering.

1. Introduction

The measurement of heat flux in extreme thermal environments is challenging. Due to the high temperature and high heating rate, surface instrumentation is difficult to maintain reliability to acquire accurate data for long time [1]. It is proposed to estimate the unknown heat flux indirectly by solving an inverse heat conduction problem (IHCP) with the knowledge of interior temperature measurements [2–4]. The techniques for the resolution of IHCPs find wide applications in the reconstruction of long-duration surface heating conditions encountered in, for instance, the spacecraft thermal protection system [5], combustion chamber [6] and high energy laser [7,8].

In long-duration heating, the heat conducts throughout the solid and the half-space assumption [9–11] is invalid. As a result, the heat conduction model with finite thickness is required. To give a well-posed direct problem, an additional boundary condition is needed besides the surface heat flux. Typically, it is set to be insulated [7,12] or heated/cooled with a known heat transfer coefficient [13] on the back surface of the finite domain. However, the insulation of the back surface will impede the heat dissipation and limit the surviving time. In addition, the convective heat transfer coefficient is inconvenient to determine in different environment. In this study, the temperature history of an interior point is regarded as the boundary condition [11,14]. Thus the quantification of back boundary condition is unnecessary.

Furthermore, a large temperature variation is always encountered in long-duration heating condition. The temperature-dependent effect of thermo-physical properties is significant. Zhou et al. [15] used the conjugate gradient method to resolve the nonlinear IHCP, but successive iterations were needed in this method. Moreover, in order to have access to linear analysis tools, the linearization of heat equation is essential. Various transformations have been proposed for linearization, which required prudent choices for specific materials due to the assumptions of properties [16]. Recently, a new technique involving time rescaling is presented by Chen et al. [17,18]. They used the temperature of sensor location to evaluate the thermal property, which held constant spatially at each time interval. Thus, the nonlinear heat conduction equation is simplified into a series of piecewise linear ones. In this study, an improved method is given by searching an optimal position for the evaluation of the thermal property, and the error induced by the linearization assumption is minimized.

Among the various techniques for the estimation of surface heat flux, many investigations focus on obtaining an explicit analytical relationship between heat flux and temperature for one-dimensional problem. Monde [11] and Woodfield et al. [12] derived an analytical solution using Laplace transform and the heat flux was estimated by approximation of temperature data with polynomials in the half-power of time. Feng et al. [19] related the front surface heat flux and temperature with the same quanti-

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Nomenclature

a_i	polynomial coefficients for thermal conductivity
b_i	polynomial coefficients for thermal diffusivity
C_k	gain coefficients
c	specific heat
d	total length of finite domain
F_1, F_2	shifting functions
k	thermal conductivity
L	coordinate of remote measurement point
N	number of terms of series expansion in Eq. (23)
P	order of polynomial fitting of thermal properties
q	surface heat flux
q_M	heat flux component over time interval $[t_{M-1}, t_M]$
r	number of future time steps
T	temperature
t	time variable
U	uncertainty
u_i	polynomial coefficients for real temperature profile
v_i	polynomial coefficients for rescaled temperature profile
x	spatial coordinate
x_m	sensor location close to heated surface
Z	temperature rise due to a unit step heat flux

Greek symbols

α	thermal diffusivity
$\Delta\tau$	dimensionless time step size
δ^2	mean squared error
ε	bias
Φ	Kirchhoff temperature
φ	transformed function
γ_n	eigenfunctions
λ_n	eigenvalues
θ	dimensionless temperature difference
ρ	density
σ^2	variance
τ	dimensionless time variable
ξ	dimensionless spatial coordinate
Ψ	temperature rise due to estimated heat flux components and change in remote boundary condition

Subscripts

0	value at initial time
L	value at $x = L$
max	maximum
opt	optimal value
mea	measured value
cal	calculated value

Superscripts

$*$	variable defined in rescaled time domain
$-$	dimensionless variable

ties on the back surface by the transfer functions in Laplace domain, which were further expanded using Hadamard Factorization Theorem and translated into time domain. Woodbury et al. [14] obtained a filter solution to the IHCP and the heat flux at any time was expressed in terms of temperatures of two internal points at only few time steps nearby. Coy [20] used a polynomial with time-dependent coefficients to fit the temperature profile of a one-dimensional domain and the surface temperature/heat flux was further approximated based on temperature measurements. In addition, the calibration methodology has recently been proposed to overcome the effects of thermocouple positioning and time constant on temperature measurement. The non-integer system identification (NISI) method gave a formulation involving fractional derivatives between heat flux and temperature with coefficients determined via calibration [21,22]. Alternatively, the calibration integral equation was derived to estimate the unknown heat flux using measured temperature and quantities acquired in a calibration run [10]. The present study utilizes the shifting function method developed by Chen et al. [13] and Lee et al. [8] to solve the heat conduction problem with inhomogeneous boundary conditions. A new relationship between surface heat flux and temperature is derived, which allows us to estimate the heat flux sequentially by using the function specification method [2].

This paper resolves a nonlinear IHCP to estimate the surface heat flux using two interior temperature measurements. A brief outline of the paper follows. First, a mathematical description of the IHCP is presented. Next, the nonlinear heat equation is linearized based on Kirchhoff transformation and time rescaling method. An optimal position for calculation of the rescaled time is obtained. Then the solution to this linearized heat equation is derived by employing the shifting function method. Afterwards, the heat flux is estimated sequentially and stabilized via function specification method. Finally, the present inverse method is illustrated by an example problem. The effect of rescaled time step on the stability of present method is also discussed.

2. Model development

2.1. Problem description

Fig. 1 shows a finite domain of thickness d with a uniform temperature distribution at initial time $t = 0$. The thermal properties is considered as temperature-dependent. The solid is insulated along the x direction to ensure the one-dimensional heat conduction. The (net) heat flux exerted on the front surface ($x = 0$) is unknown and to be estimated by using the interior temperatures measured by two in-depth probes (usually the thermocouples) mounted at $x = x_m$ and L. The probe at $x = x_m$ is placed close to the heated surface to bring enough sensitivity to the change of the heat flux. The temperature history at $x = x_m$ is incorporated into a least-squares function for estimation, while the remote one ($x = L$) is used as the boundary condition of heat equation.

Consider the one-dimensional nonlinear heat conduction in the domain $x \in [0, L]$ governed by the following equation

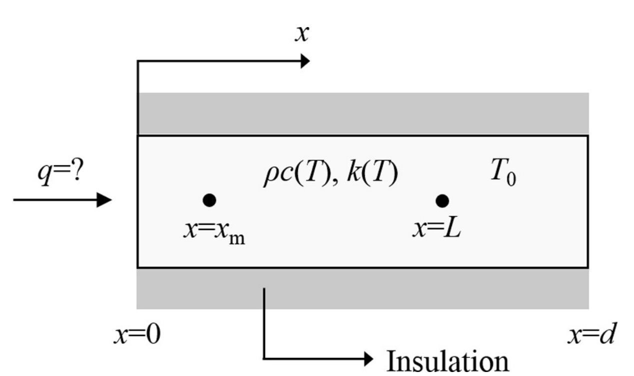

Fig. 1. Schematic of the one-dimensional nonlinear inverse heat conduction problem.