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A modified space marching method using future temperature measurements for transient nonlinear inverse heat conduction problem

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ABSTRACT

A new modified space marching method is presented for solving transient nonlinear inverse heat conduction problems. The new method combines the merits of space marching and function specification methods. Future temperature measurements are used in a least square manner to stabilize the calculation results. The transient surface temperature and heat flux can be accurately recovered by the method without employing iteration procedure or data smoothing technique. Three types of heat flux profiles were used to evaluate the proposed inversion method and a comparison to the classical space marching method was drawn to demonstrate the advantage of the present method. A numerical experiment is performed to investigate the influence of parameters in the method as well as to illustrate the selection of optimal parameters. The present method calculates surface temperature and heat flux values in a sequential manner therefore is suitable for on-line estimation application.

1. Introduction

The inverse heat conduction problem (IHCP) involves determining the transient surface heat flux using measured temperature history inside a heat conduction body [1]. It has a wide engineering application such as the manufacturing process monitoring [2] and propellant combustion design [3]. The solution of IHCPs is much more challenging than the corresponding direct problems as IHCPs are mathematically ill-posed and a small error in measured temperatures would be amplified in calculated heat flux results [4]. Particularly, when the thermal properties of solids are temperature dependent, then the IHCPs becomes nonlinear ones, which are more difficult to solve than linear IHCPs [5].

For linear IHCPs, a variety of numerical solution methods have been developed including function specification (FS) [6], Tikhonov regularization (TR) [7], space marching [8,9], gradient optimization [10–12], and intelligent optimization method [13–15]. Among these methods, the FS method calculates heat flux values in a sequential manner, which is suitable for on-line application. The influence of temperature measuring error is reduced by the use of several future temperature measurements [6]. The space marching method is based on finite difference algorithm which extrapolates the temperature field from direct region to the unspecified boundary [16]. The surface heat flux is calculated after the complete temperature field is obtained. The TR method and gradient optimization method calculate all the heat flux components at one time, therefore more storage memories are required in practical use.

For nonlinear IHCPs, one may encounter difficulty in calculating the sensitivity coefficient involved in different solution methods, because the solid properties are dependent on temperatures. To overcome this difficulty, Beck [17] adopted an iteration procedure to calculate heat flux with FS method. He [5] also linearized the nonlinear IHCPs by assuming constant properties and sensitivity coefficient from one time step to the next. In addition, Raynaud and Bransier [16] proposed a modified space marching method which employed future temperatures for nonlinear IHCPs. The used future temperatures had a same number as the spatial grid points in inverse region. Taler [18] presented a new space marching method for nonlinear IHCPs. The fixed-point iteration procedure and temperature data smoothing technique were involved in his method. More recently, Daouas and Radhouani [19] adopted extended Kalman filter algorithm along with fixed-interval smoothing technique to solve a transient nonlinear IHCP. Beck [2] developed the filter solutions for nonlinear IHCPs and the temperature dependent filter coefficients were obtained by interpolating filter coefficients at discrete temperatures. Cui et al. [4,20,21] introduced the complex variable differentiation method in the calculation of sensitivity coefficient for nonlinear IHCPs. The temperature dependent solid properties and multiple parameters in

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Nomenclature

c specific heat of the heat conduction body, $J/(kg \cdot K)$
F_i & F'_i (i = 1, 2, 3) coefficients in energy balance equations, all dimensionless except F'_1 in $K \cdot m^2/W$
k time index
n number of spatial grids in inverse region
N number of spatial grid points in inverse region
NT total number of discretized temperature or heat flux values
q surface heat flux, W/m^2
q_N The maximum value of heat flux, W/m^2
r_T & r_q number of future temperatures in two stages
s_T & s_q root mean square error of surface temperature and heat flux, dimensionless
t time, s
T temperature, K
x spatial coordinate, mm
Δx spatial step in direct calculation, mm
Δx_{in} spatial step in inverse calculation, mm
Δt time step in direct and inverse calculation, s
ΔT temperature increment within one time step, K

Greek
ρ density of the heat conduction body, kg/m^3
λ thermal conductivity of the heat conduction body, $W/(m \cdot K)$

Subscript
L total length of the heat conduction body
m temperature sensor location
i spatial index
0 initial temperature
exact exact values
cal calculated values

Superscript
j time index
$\sim$ measured temperature

functional forms of heat flux can be effectively recovered with their method.

The main objective of this paper is to present a new modified space marching method for transient nonlinear IHCPs. The proposed method combines the merits of classical space marching method and FS method. On one hand, the space marching method can deal with fully nonlinear IHCPs by finite difference algorithm but the oscillation of solution is usually significant. On the other hand, the FS method reduces oscillation of solution by using future temperatures in a least square manner, but the FS method is based on Duhamel's theorem [6] which is not originally for nonlinear problems. The combination of both methods can readily deal with fully nonlinear IHCPs without the need of iteration procedure. Moreover, the inverse solution is stabilized by using future temperature measurements thus prior smoothing of temperature data is not required. Lastly, the new method calculates surface heat flux in a sequential manner therefore can be applied in on-line calculation.

The outline of this paper is as follows. Section 2 describes the selected inverse problem considered in this paper. The algorithm formulation of the proposed modified space marching method is presented in Section 3. The performance and parameter influences of the proposed method are presented in Section 4. The summary and conclusions of this paper are finally presented in Section 5.

2. Description of inverse problem

The heat conduction model [19] to be considered is shown in Fig. 1. The conduction body is a one-dimensional slab with a length of 15 mm. The left boundary is subjected to an unknown transient heat flux denoted by q(t) and the remaining three boundaries are insulated. The inside temperatures are measured by a sensor placed at 2 mm from left boundary. The slab has temperature dependent properties described by

$$\begin{cases} \rho = 7916.758 \quad kg/m^3 \\ c(T) = (0.9723T + 188.233) \quad J/(kg \cdot K) \\ \lambda(T) = (0.02978T + 5.767) \quad W/(m \cdot K) \end{cases} \quad (1)$$

where ρ is density, c is specific heat and λ is thermal conductivity. The temperature varying range in Eq. (1) is 256–417 K.

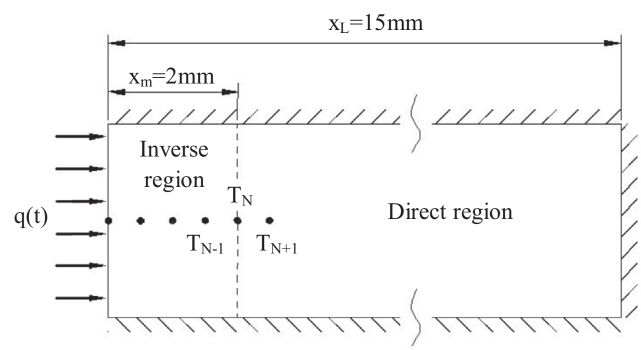

Fig. 1. Schematic of heat conduction model.

The slab has a uniform initial temperature being 293 K and the heat flux q(t) is applied between time interval 0–15 s. Consequently, the heat conduction in the slab is governed by

$$\begin{cases} \rho c(T)\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left[\lambda(T)\frac{\partial T}{\partial x}\right], & 0 < x < x_L, t \in (0, 15] \\ T(x) = T_0, & 0 < x < x_L, t = 0 \\ -\lambda(T)\frac{\partial T}{\partial x} = q(t), & x = 0, t \in (0, 15] \\ \frac{\partial T}{\partial x} = 0, & x = x_L, t \in (0, 15] \end{cases} \quad (2)$$

where T_0 is the initial temperature and the thermal properties are calculated using Eq. (1). It is noted that the right boundary of slab can follow other types of boundary condition other than the insulated one. The calculation would be no different for other boundary conditions. Now, the objective of the inverse problem is to recover the unknown heat flux q(t) using the measured temperature data at x_m in Fig. 1.

3. Algorithm formulation

In space marching method the computational domain is divided into two types of subdomains, namely inverse region and direct region (Fig. 1). The two boundaries of direct region are specified with the measured temperature data, thus the temperature field in direct region can be calculated by any numerical procedure with