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Mixed convection in a vertical flat microchannel

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ABSTRACT

The paper presents results of an investigation into mixed convection in a vertically oriented microchannel with slip boundary conditions. Solutions of the problem were obtained analytically and using a numerical approach based on the Lattice Boltzmann method (LBM). The solution yields relations, which enable estimating velocity and temperature profiles and the Nusselt number as functions of the Knudsen, Rayleigh and Prandtl numbers. It was shown that Knudsen number effects are prevailing in the vicinity of the wall, whereas near the centerline of the channel effects of the Rayleigh number are stronger. If the Rayleigh numbers are high, velocity profiles demonstrate M-shapes with a point of minimum at the channel centerline, whereas temperature profiles flatten so that the fluid temperature in the channel cross-section tends to the wall temperature. The temperature jump magnitude on the wall is dependent on the Prandtl number value and decreases with the increasing Prandtl numbers. For almost all combinations of the parameters considered in this paper, higher Knudsen numbers entail heat transfer deterioration except for the case of Pr = 10 and Ra = 100. Increasing the Knudsen number diminishes hydraulic resistance for low Rayleigh numbers, but for high values of the Rayleigh numbers the trend is reversed. It was shown that mixed convection in microchannels can be successfully simulated using the LBM methodology, whose deviation from the analytical solution and is less than 1%.

1. Introduction

The interest to modelling of microchannel flows both from scientific and practical points of view has permanently grown over the last decades being inspired by the development of various microelectromechanical and microenergy systems, micro- and nanofabrication, as well as nanotechnology [1–4]. Microdevices deliver examples of the geometries, where measurements and modeling involve microscales whose characteristic length varies in the range of the micrometers. The scaling laws used in the practice of commonly known mechanical engineering, turbomachinery etc. often contradict to the real fluid flow and heat transfer phenomena in gas flows in microsystems affected not by the rarefaction of the gases due to the reduced density, but by the reduced length scales of the flow geometry itself [5].

The effects of rarefaction in flows in microchannels are described by the Knudsen number Kn, a dimensionless criterium proportional to the ratio of the mean free path of a gasmolecule, L, and a characteristic length of the channel cross section L_{ref}. The exact definition of the Knudsen number employed in the present work is presented below in the section "Governing equations". The Knudsen number characterizes the effects of slippage at the surface. Its value describes the velocity and temperature jump at the wall (see the slip boundary conditions in the section "Governing equations" below). In addition, the Knudsen number includes the viscous slip coefficient (see the definition of the Knudsen number in the section "Governing equations"). This coefficient takes into account the processes of accommodation on the surface. The modern theory of the viscous slip coefficient can be found in the works of Sharipov and Seleznev [6], Sharipov [7], and Agrawal and Prabhu [8]. Thus, one can say that the Knudsen number enables describing sophisticated fluid mechanics and thermal processes that take part on the fluid-solid interface in microflows.

For $\mathrm{Kn} \leqslant 10^{-2}$, fluid flow can still be simulated using the Navier–Stokes equations complemented with no-slip boundary conditions [9]. For $10^{-2} \leqslant \mathrm{Kn} \leqslant 10^{-1}$, the slip-flow regime arises. Strictly saying, these limits are not filly confidently defined so far, so that for instance the range of $10^{-3} \leqslant \mathrm{Kn} \leqslant 10^{-1}$ for slip flow conditions was defined in Ref. [10]. The transition regime arises for $10^{-1} \leqslant \mathrm{Kn} \leqslant 10$; in this case the continuum assumption is not valid any more. Such flows can be modeled using simulation tools

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Nomenclature

a	half of the channel width
A	axial wall temperature gradient
c	molecular velocity
c_p	specific heat at constant pressure
d	space dimension
D_e	hydraulic (equivalent) diameter
F	external force term
f	distribution function
g	gravitational acceleration
G	Archimedes force
$\mathbf{k}$	unit vector directed streamwise
L	free path of a gas molecule
p	pressure
q	heat flux
R	gas constant
t	time
T	temperature
u	axial velocity component
$\mathbf{u}$	velocity vector
x, y	Cartesian coordinates

Greek symbols

α	thermal diffusivity
β	coefficient of thermal expansion
γ	viscous slip coefficient
λ	friction factor
μ	dynamic viscosity
Π	channel perimeter
ρ	density
τ	relaxation time
ϕ	relative temperature

Dimensionless values

$\tilde{f}$	dimensionless distribution function
E	parameter of pressure gradient
N_y	number of elements in the lattice column across the channel
Nu	Nusselt number
Kn	Knudsen number
Ra	Rayleigh number
Re	Reynolds number
Pr	Prandtl number
$\tilde{u}$	lattice dimensionless velocity
U	dimensionless axial velocity
Y	dimensionless coordinate
$\boldsymbol{\Theta}$	dimensionless temperature
$\tilde{\rho}$	dimensionless density
$\tilde{\tau}$	dimensionless relaxation time

Subscripts

0	case Ra = 0
00	case Ra = Kn = 0
j	velocity directions
T	energy
w	wall
$*$	incompressible medium

Superscripts

e	equilibrium distributions

Acronyms

BGK	Bhatnagar-Gross-Krook
CFD	Computational Fluid Dynamics
LBM	Lattice-Boltzmann method
2D	two-dimensional

involving direct Monte Carlo simulation; here the Burnett equations [11] become necessary.

The heat flux through a solid–gas interface in microchannels generates a temperature jump, which can be defined as [12]

$$\Delta T_i \sim L\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{1}$$

where L is proportional to l_k, which is the interfacial thermal resistance length known also as Kapitza length discovered first in 1941 [13].

Neumann and Rohrmann [14] simulationed flows in the finite Knudsen range for slip and transition flows based on the Lattice Boltzmann method within the Peano framework. Afterwards the Lattice Boltzmann solver was applied to a microreactor consisting of differently sized channels and a reactor chamber. The authors involved a simple Bhatnagar-Gross-Krook (BGK) collision kernel in coarse grid regions located rather far from the slip boundaries. The obtained results agree well with the theory and non-adaptive simulations.

Avramenko at al. [15–17] modeled isothermal steady-state and start-up flows in straight and curved microchannels using both analytical and LBM methodology. The latter proved to be in excellent agreement with the analytical solution. Effects of the Knudsen number and channel curvature on the velocity profiles and hydraulic resistance of the channel were studied in details.

Numerical simulations of Avramenko at al. [15–17] have been performed with the help of the LBM methodology under isothermal conditions without taking into account the energy equation. In the current study, heat transfer was modeled using the second set of distribution functions introduced specially for this purpose. Lattice Boltzmann equations for the fluid flow and thermal problems are coupled with each other due to the presence of the term taking into account the Archimedes force. The dimensions of the grids used for both equations are the same, but the relaxation times used in computations of the momentum and energy are different. Also, for the lattice used to calculate the temperature, separate boundary conditions were set.

Problems of combined free and forced convection emerged heat transfer in channels have been studied for many decades. Various geometries and influential factors have been investigated both analytically and numerically. Galanis and Behzadmehr [18] performed a review of studies on mixed convection in vertical ducts overviewing the time period 20 years backwards from the year 2008. The review represents results of experimental, analytical and numerical investigations demonstrating complexity of such flows in comparison with pure forced convection flows. Different is the need to modify velocity and temperature profiles, to take into account possible flow reversal at different cross-sections, as well as noticeably buoyancy effects on the flow regime, surface heat transfer and friction.

In a recent review, Dawood et al. [19] discussed the last achievements in studies of free, forced and mixed convection in

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