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Similarity solutions for the evolution of polydisperse droplets in vortex flows

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ABSTRACT

A new mathematical analysis of the dynamics of evaporating sprays in the vicinity of a vortex flow field is presented. The governing equations for a polydisperse spray evaporating in an unsteady viscous vortex flow are formulated using the sectional approach. First, new similarity solutions are found for the dynamics of the spray in a mono-sectional framework. It is shown that similarity for the droplets' drag term exists, and an explicit model for the drag is found using perturbation theory. Numerical simulations are conducted to validate the main assumptions of the analytic approach adopted in this study. An extension of the mono-sectional solution of the spray equations to a polydisperse spray solution is then derived and the dynamics of polydisperse spray in an Oseen type vortex are presented. It is shown that for a given radial location, the droplets in each section reach a maximal radial velocity due to the effect of vorticity. A simple model is derived for the prediction of this maximal radial velocity of the droplets using perturbation theory, which agrees very well with the full similarity solution. The present study shows that spray dynamics is highly affected by the droplets' size, but also by the spray initial size distribution, even when the same Sauter mean diameter is considered. This may have far reaching implications, especially in spray combustion applications.

1. Introduction

The dynamics of evaporating sprays in vortical environments play an important role in many practical combustion applications such as gas-turbines and swirl combustion chambers. The interactions between gaseous diffusion flames and a vortex flow-field were extensively studied (Alain and Candel (1988); Marble (1985); Renard et al. (2000)). In a previous study of Dagan et al. (2015), unsteady turbulent spray-flame instability in a concentric jet combustion chamber was studied using large eddy simulations (LES). Droplet grouping and ligament structures were identified in the vicinity of vortices in large recirculation zones. Such droplet flow patterns were shown to have a significant role in the evolution of large-scale turbulent flames compared to that of gaseous turbulent flames. Using direct numerical simulations (DNS) of Diesel spray combustion in the vicinity of a recirculation zone, Shinjo and Umemura (2013) showed that when droplets are larger than the Kolmogorov microscale, mixing is strongly enhanced by the presence of droplets and fuel vapor clusters are likely to form quickly when the droplet number density is high. They suggested that external group combustion is likely to occur near the recirculation zone.

The effects of droplet clustering on evaporation were thoroughly discussed by Sirignano (1999) and Harstad and Bellan (2001). Clusters of droplets are formed in vortical flows, as the droplets accelerate towards the outer region of the vortex (Bellan and Harstad (1991)). The dynamics of droplet-vortex interactions and their influence on the structure of an evaporating spray was numerically studied by Park et al. (1996). Droplet-vortex interaction in the Karman-vortex street was studied by Burger et al. (2006), using DNS and a theoretical approach imposing a harmonically oscillating flow field. Recently, Franzelli et al. (2014) numerically characterized the regimes of spray flame-vortex interactions. They used a two-dimensional Oseen type vortex in their study. However, their study relates to a vortex moving perpendicularly through an opposed flow spray sheet. These studies emphasize the need for a more fundamental understanding of the effects of vortical environments and recirculating flows on droplet dynamics, which appear in turbulent, as well as laminar environments.

The objective of the present work is to analytically study the influence a vortex flow-field has on evaporating polydisperse sprays.

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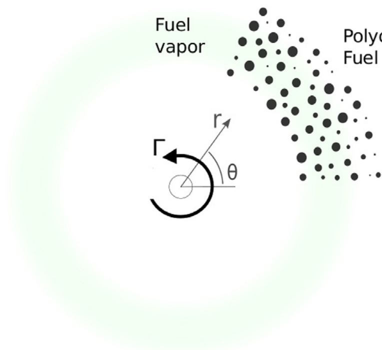

Fig. 1. Schematic description of polydisperse spray in a vortex flow field. Droplets are partially presented in a section for clarity.

A new mathematical formulation for the dynamics of polydisperse sprays in the vicinity of a vortex flow field is presented. The governing equations for a polydisperse spray evaporating in an unsteady vortex are studied and new similarity solutions are found for the dynamics of the spray, using the sectional approach. Extensive use has been made of the sectional approach in a variety of polydisperse sprays in both nonreacting and reacting systems. This approach was shown to produce excellent agreement with experimental data of different spray systems (Greenberg (2007); Greenberg et al. (1993); Katoshevski and Tambour (1993); Massot et al. (1998); Tambour (1985)). In the sectional approach framework the droplet size distribution is divided into size-sections and an averaging technique is applied within each section, whereby the sectional conservation equations are derived.

It should me mentioned that several studies (see for example De Chaisemartin, 2009; De Chaisemartin et al., 2009; Vié et al., 2015) showed the limitation of the sectional approach in reproducing correctly the dynamics of inertial enough particles (i.e particles with high enough Stokes numbers) in multi-vortices configurations. Thus, in the context of vortical flows, the use of the sectional approach is valid for either droplets with relatively small Stokes numbers or for specific configurations, such as the single vortex configuration of the present manuscript.

The configuration under consideration is schematically illustrated in Fig. 1. Initially, an axisymmetric cloud of polydisperse droplets is introduced. A single vortex of strength Γ is introduced at $t = 0$, located at $r = 0$. Drag forces accelerate the droplets about the center of the vortex.

In Section 2, the governing equations for polydisperse sprays in vortex flow are presented. Next, analytical solutions for a mono-sectional spray are obtained. Then, a unique extension of the mono-sectional solutions to a polydisperse case are derived. Finally, solutions for the dynamics of polydisperse sprays are presented in Section 5. Despite the simplicity of the current model, its predictions provide insight into the driving mechanisms behind the much more complex turbulent, polydisperse spray regime, in which similar droplet dynamics occur.

2. Governing equations

2.1. Gas phase

The equations for the general case of a polydisperse spray evaporating in a two-dimensional unsteady axisymmetric vortex flow are presented below. A polar coordinate system (r, θ) is employed in the following derivation of the equations. As a result of our assumption of an axisymmetric flow, all derivatives with respect to the angular coordinate θ are assumed zero. A constant density for the host gas is assumed.

The radial and tangential gas-phase momentum equations are given by

$$\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} - \frac{v_\theta^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right) - \sum_{j=1}^{N_s} \tilde{Q}_j F_{r,j} - \dot{S}_{ev,r} \tag{1}$$

$$\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_r v_\theta}{r} = \nu \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right) - \sum_{j=1}^{N_s} \tilde{Q}_j F_{\theta,j} - \dot{S}_{ev,\theta} \tag{2}$$

where ρ is the gas-phase density, v_r is the gas radial velocity, v_θ is the tangential velocity of the host gas, ν is the kinematic viscosity and p is the pressure. $\tilde{Q}_j$ denotes mass fraction of the liquid fuel in size section j and N_s is the total number of sections. $\dot{S}_{ev}$ accounts for the momentum transferred to the host gas by the vapors and will be written explicitly in the momentum equations for the spray. $F_{r,j}$ and $F_{\theta,j}$ describe the interaction between the gas phase and the droplets of size section j in the radial and tangential directions, respectively, being proportional to the relative velocity between the droplets and the gas

$$F_{r,j} = \tau_j^{-1}(v_r - u_{r,j}); \quad F_{\theta,j} = \tau_j^{-1}(v_\theta - u_{\theta,j}) \tag{3}$$

where $u_{r,j}$ and $u_{\theta,j}$ are the radial and tangential velocities of the droplets of section j, respectively. τ_j is the sectional droplet relaxation time-scale. Terms in Eqs. (1) and (2) that are responsible for the interactions between the gas and the liquid phase, namely F and S_{ev}, are presented here only for the sake of generality and will eventually be omitted from the gas momentum equations, since for the evaporating sprays considered here, their effect on the velocity field is negligible (see Katoshevski and Tambour, 1993; 1995 and Zhu and Tambour, 1994).

2.2. Liquid spray phase

In the sectional approach framework, the multi-size droplet population is presented by a set of N_s sectional conservation equations of the form

$$\frac{\partial Q_j}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(r u_{r,j} Q_j) = -C_j Q_j + B_{j,j+1} Q_{j+1}; \quad j = 1, 2, \ldots, N_s \tag{4}$$

where the coefficient $B_{j,j+1}$ accounts for droplets from section $(j+1)$ which are added to section j during their evaporation, whereas C_j accounts for evaporation of droplets within section j and for droplets that move from section j to section $(j-1)$. The spray sectional momentum equations are

$$\frac{\partial u_{r,j}}{\partial t} + u_{r,j}\frac{\partial u_{r,j}}{\partial r} - \frac{u_{\theta,j}^2}{r} = F_{r,j} + \frac{1}{\tilde{Q}_j}(S_{j,r}^{L,M} - u_{r,j} S_j) \tag{5}$$

$$\frac{\partial u_{\theta,j}}{\partial t} + u_{r,j}\frac{\partial u_{\theta,j}}{\partial r} + \frac{u_{r,j} u_{\theta,j}}{r} = F_{\theta,j} + \frac{1}{\tilde{Q}_j}(S_{j,\theta}^{L,M} - u_{\theta,j} S_j) \tag{6}$$

The last term in the brackets on the RHS of Eqs. (5) and (6) represents loss of linear momentum $(u_j S_j)$ due to evaporation of droplets in section j, and the linear momentum (S_j^{LM}) added to section j due to droplets from higher sections that are added to section j during their evaporation (see Katoshevski and Tambour, 1993). Here,

$$S_j = -C_j \tilde{Q}_j + B_{j,j+1} \tilde{Q}_{j+1} \tag{7}$$

and

$$S_{j,r}^{L,M} = -C_j \tilde{Q}_j u_{r,j} + B_{j,j+1} \tilde{Q}_{j+1} u_{r,j+1} \tag{8}$$

$$S_{j,\theta}^{L,M} = -C_j \tilde{Q}_j u_{\theta,j} + B_{j,j+1} \tilde{Q}_{j+1} u_{\theta,j+1} \tag{9}$$