International Journal of Multiphase Flow 97 (2017) 78–93

Contents lists available at ScienceDirect

International Journal of Multiphase Flow

journal homepage: www.elsevier.com/locate/ijmulflow

Closure relations effects on the prediction of the stratified two-phase flow stability via the two-fluid model

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ARTICLE INFO

Article history:
Received 6 April 2017
Revised 5 July 2017
Accepted 7 July 2017
Available online 11 July 2017

Keywords:
Two-fluid model
Stratified flow
Closure relations
Stability
Sheltering
Multiple solutions

ABSTRACT

The possibility of predicting the exact long wave linear stability boundary via the two-fluid (TF) model for horizontal and inclined stratified two-phase flow is examined. The application of the TF model requires the introduction of empirical closure relations for the velocity profile shape factors and for the wave induced wall and interfacial shear stresses. The latter are recognized as the problematic closure laws. In order to explore the closure relations effects and to suggest the necessary modifications that can improve the stability predictions of the TF model, the results are compared with the exact long wave solution of the Orr–Sommerfeld equations for the two-plate geometry. It is demonstrated that with the shape factors corrections and the inclusion of wave induced stresses effects, the TF model is able to fully reproduce the exact long wave neutral stability curves. The wave induced shear stresses in phase with the wave slope, which give rise to the so called "sheltering force", were found to have a remarkable destabilizing effect in many cases of horizontal and inclined flows. In such cases, the sheltering effects must be included in the TF model, otherwise the region of smooth stratified flow would be significantly over predicted. Based on the results of the exact analysis, a simple closure relation for the sheltering term in the TF model is provided.

1. Introduction

Stratified flow is a basic two-phase flow pattern where a continuous layer of a light phase flows on top of a heavier phase. Various important chemical and industrial processes, frequently encounter horizontal and inclined gas–liquid and liquid–liquid stratified two-phase flows. Under certain operating conditions however, interfacial instabilities can arise and may produce undesired effects and trigger flow pattern transition. The exploration of the interface stability is therefore of practical importance. Additionally, it may also rule out the feasibility of part of the solutions in the multiple holdup solution regions that characterize inclined stratified flows.

Exact solutions for steady laminar stratified flow in inclined pipes are available in the literature (e.g. Ullmann et al., 2004; Goldstien et al., 2015). However, exact formulation of transient stratified flow in pipe geometry is too complicated for conducting a rigorous stability analysis. A widely used approach to study the flow stability is to perform a stability analysis on the simplified one dimensional transient two-fluid (TF) model, where long-wave is an inherent assumption (e.g., Lin and Hanratty, 1986; Andritsos et al., 1989; Barnea and Taitel, 1994; Brauner and Moalem Maron, 1992a). Since the velocity profiles are not resolved in the framework of TF analysis, the application of the TF model requires the introduction of empirical closure relations. These are needed for the velocity profile shape factors (to represent correctly the inertia terms) and for the wall and interfacial shear stresses (e.g., Brauner and Moalem Maron, 1993). The models adopted for closure ought to bridge the gap between the details of the pertinent hydrodynamic phenomena and the macro-averaged representation of the flow. To circumvent the problem, plug flow has been commonly assumed (shape factors of 1). For the wall and interfacial shear stresses, the simplest and most common approach is to use quasi-steady shear stress models (e.g., Brauner and Moalem Maron, 1992b; Barena and Taitel, 1994). Consequently, in the stability analysis only the components of the stresses in phase with the wave height are considered, in accordance with the K-H mechanism. Indeed, in the literature of gas–liquid and liquid–liquid flows, the instability of the stratified flow pattern is commonly associated with the Kelvin-Helmholtz (K-H) mechanism. This mechanism attributes the growth of interfacial disturbances to the inertia forces, which

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http://dx.doi.org/10.1016/j.ijmultiphaseflow.2017.07.010

give rise to wave induced pressure fluctuations in phase with the wave height.

The ability of the averaged TF model to predict macroscopic behavior is however largely dependent on a successful representation of the interactions between the basic flow and the wave induced hydrodynamics by the closure laws used. The introduction of simplistic empirical closure relations may lead to inherent inaccuracies in the TF model predictions. It has long been recognized that a reasonable prediction of the smooth-stratified flow boundary in gas–liquid flows requires incorporation of an interfacial shear stress term in phase with the wave slope (see reviews by Hanratty (1991) and Brauner and Moalem Maron (1996)). The origin of this term was attributed to the Jeffreys (1925), Benjamin (1959) and Miles–Phillips mechanisms (e.g., Miles, 1959, 1962) for wind generated waves. According to those mechanisms, wind-wave interactions result in an interfacial stress component in phase with the wave slope, which is essential to enable energy transfer from the wind to the wave. Waves grow when the energy input from the wind exceeds the viscous dissipation in the waves. In Jeffreys' criterion for instability, the resulting critical gas velocity is proportional to the liquid viscosity, and an empirical sheltering coefficient was introduced, which was tuned to fit data of the critical wind velocity. This criterion was applied to predict the stratified-smooth/stratified-wavy (SS/SW) transition in horizontal gas–liquid pipe flows (e.g., Taitel and Dukler, 1976; Andritsos et al., 1987). Upon including a shear stress component in phase with the wave slope in the TF closure relation for the interfacial shear, a generalized stability criterion was obtained (Brauner and Moalem-Maron, 1993, 1994). The resulting criterion combines the so-called 'viscid K-H' mechanism (due to the inertia terms of both of the phases) and the so- called 'sheltering' mechanism. Accordingly, both mechanisms can have a role in determining the critical conditions for the SS/SW transition and should be considered (e.g., Lopez de Bertodano et al., 2013).

Recently, it has been shown by Kushnir et al. (2014) that the interaction between the wave and the flow field in the two layers gives rise to wave induced stresses in phase with the wave slope. Expressions for such stresses were derived, for a two-plate geometry and long wave disturbances showing that they should not be ignored in the framework of long wave stability analysis. Consequently, the interface instability is affected also by the shear components in phase with the wave slope (so-called 'sheltering mechanism'), and not only by the inertia of the two phases that are responsible for the K-H mechanism.

This study has been undertaken with the purpose of exploring the linear stability of two-phase stratified flows by the TF model. The main purpose is to determine quantitatively the effects of the closure relations on the TF model stability prediction. In particular, the effects of wave induced stresses that are in phase with the wave slope (i.e., the sheltering term) are explored. To this end, the results are compared with the exact long-wave solution of the Orr–Sommerfeld equations for a two-plate geometry. Such a study can reveal to what extent the 'sheltering mechanism' is of importance in destabilizing the interface between two laminar layers in the framework of TF models. The study can also identify the modifications that are needed for improving the TF model predictions for more complicated and practical geometries where exact solutions are not available.

2. Statement of the problem

Consider the laminar flow of two immiscible, incompressible fluids, labeled j=1, 2, flowing in an inclined channel ($0 \le \beta \le \pi/2$) as shown in Fig. 1. The flow, assumed isothermal and two dimensional, is driven by an imposed pressure gradient and a component of gravity in the $\hat{x}$ direction. We restrict our attention to stable density stratification, so that the heavier fluid always forms the lower layer. The basic (steady and fully developed) flow is a two-layered Poiseuille flow with a flat interface, and the corresponding non dimensional velocity profiles and pressure gradient are given by

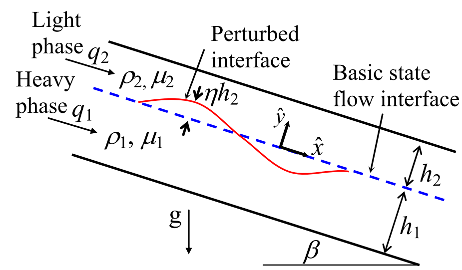

Fig. 1. Schematic description of two-layer flow configuration in an inclined channel.

$$u_j \equiv U_j = 1 + a_j y + b_j y^2, v_j = 0, j = 1, 2 \tag{1a}$$

$$\frac{dp_j}{dx} \equiv \frac{dP}{dx} = \frac{2}{\mathrm{Re}_2}\left(\frac{(1+nr)\tilde{Y} - (m+n)(1-r)}{n(1+n)(1-r)}\right) \tag{1b}$$

where

$$a_1 = \frac{a_2}{m}, a_2 = \frac{m - n^2 + n\tilde{Y}}{n^2 + n} \tag{2a}$$

$$b_1 = -\frac{m + n - \tilde{Y}}{(n^2+n)m}, b_2 = -\frac{m + n + n\tilde{Y}}{n^2 + n} \tag{2b}$$

$$\tilde{Y} = \frac{n(1-r)\mathrm{Re}_2 \sin\beta}{2Fr_2} \tag{2c}$$

The dimensionless variables and parameters are defined as follows

$$(u_j, v_j) = (\hat{u}_j, \hat{v}_j)/u_i,\ (x, y) = (\hat{x}, \hat{y})/h_2,\ \ p_j = \hat{p}_j/\rho_2 u_i^2$$
$$m = \frac{\mu_1}{\mu_2}, r = \frac{\rho_1}{\rho_2}, n = \frac{h_1}{h_2}, \mathrm{Re}_2 = \frac{\rho_2 u_i h_2}{\mu_2}, Fr_2 = \frac{u_i^2}{g h_2} \tag{3}$$

where $\hat{u}_j$, $\hat{v}_j$, and $\hat{p}_j$ are the velocities components and pressure of phase j fluid, μ_j and ρ_j are the corresponding dynamic viscosity and density, respectively, and g denotes the gravitational acceleration. The velocity u_i represents the basic flow interface velocity, and h_j is the basic flow layer height of phase j fluid (see Fig. 1).

The Martinelli and inclination parameters, which are the common dimensionless parameters for two-phase flow and can be directly calculated by the specified fluids' properties and operational conditions, are define as follows

$$X^2 \equiv \frac{(-d\hat{P}/d\hat{x})_{1s}}{(-d\hat{P}/d\hat{x})_{2s}} = mq, \quad q = \frac{q_1}{q_2}, \quad Y \equiv \frac{\rho_2(r-1)g\sin\beta}{(-d\hat{P}/d\hat{x})_{2s}} \tag{4}$$

Here q_j is the feed flow rate of phase j and $(-d\hat{P}/d\hat{x})_{js} = 12\mu_j q_j/H^3$ is the corresponding superficial pressure drop for single phase flow in the channel, where $H = h_1 + h_2$. Given the parameters m, Y, and X^2 (or q), the heavy phase holdup, $\tilde{h} = h_1/H = n/(1+n)$, is obtained by solving the following implicit algebraic equation (e.g., Ullmann et al., 2003a)

$$Y - \frac{mq(1-\tilde{h})^2\left[(1+2\tilde{h})m + (1-m)\tilde{h}(4-\tilde{h})\right] - \tilde{h}^2\left[(3-2\tilde{h})m + (1-m)\tilde{h}^2\right]}{4\tilde{h}^3(1-\tilde{h})^3\left[\tilde{h} + m(1-\tilde{h})\right]} = 0 \tag{5}$$