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Hydrodynamic force and torque models for a particle moving near a wall at finite particle Reynolds numbers

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ABSTRACT

This research work is aimed at proposing models for the hydrodynamic force and torque experienced by a spherical particle moving near a solid wall in a viscous fluid at finite particle Reynolds numbers. Conventional lubrication theory was developed based on the theory of Stokes flow around the particle at vanishing particle Reynolds number. In order to account for the effects of finite particle Reynolds number on the models for hydrodynamic force and torque near a wall, we use four types of simple motions at different particle Reynolds numbers. Using the lattice Boltzmann method and considering the moving boundary conditions, we fully resolve the flow field near the particle and obtain the models for hydrodynamic force and torque as functions of particle Reynolds number and the dimensionless gap between the particle and the wall. The resolution is up to 50 grids per particle diameter. After comparing numerical results of the coefficients with conventional results based on Stokes flow, we propose new models for hydrodynamic force and torque at different particle Reynolds numbers. It is shown that the particle Reynolds number has a significant impact on the models for hydrodynamic force and torque. Furthermore, the models are validated against general motions of a particle and available modeling results from literature. The proposed models could be used as sub-grid scale models where the flows between particle and wall can not be fully resolved, or be used in Lagrangian simulations of particle-laden flows when particles are close to a wall instead of the currently used models for an isolated particle.

1. Introduction

The wall-bounded particle-fluid two-phase flows exist widely in numerous industrial and natural processes, such as the solid-fluid flow in industrial pneumatic conveying (Laín and Sommerfeld, 2012), the flows in a pump, the flows in fluidized bed (Capecelatro et al., 2014; Lu et al., 2013), sediment deposition and transport in rivers (Kidanemariam et al., 2013). One of the crucial phenomena in such flows is the interaction between particles and a solid wall.

This research work is basically interested in the motion of a single spherical particle close to a solid wall in fluid flow at finite particle Reynolds numbers. It is an elemental process in the particle-fluid two-phase flows and an important ingredient in treating boundary condition in numerical simulation of such flows. The particle-resolved direct numerical simulation (PR-DNS) has been emerging as a powerful research tool for particle-fluid two-phase flows. It can be used to track the motion of particles, fully resolve the surrounding fluid flows and calculate the hydrodynamic forces and torques acting on each particle (Ladd, 1994b; Lucci et al., 2010; Shao et al., 2012; Uhlmann, 2008; Uhlmann and Doychev, 2014; Wang et al., 2013; 2010). To numerically simulate the wall-bounded two-phase flow, it is necessary to generate grids of finite-size in the flow domain. When the gap between the particle and the wall is smaller than one or two grid sizes, neither the fluid motion in the gap nor the force and torque on the particle can be solved accurately (ten Cate et al., 2002). Under this condition, the hydrodynamic viscous force and torque, which arises from the high pressure when the interstitial fluid is squeezed out of the small space between the two close solid surfaces, should be compensated by establishing appropriate sub-grid scale models (Nguyen and Ladd, 2002; Zhang et al., 2005).

There are two types of sub-grid scale models used to compensate the hydrodynamic force and torque on the particle when the gap is less than one or two grids. The first one is the method of potential force (Ozdemir et al., 2010; Wang et al., 2008), which means that the resistance on the particle is set to the function of gap size between the particle and the wall. The second one is the method

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of lubrication force (Dance and Maxey, 2003; Portela and Oliemans, 2003). The asymptotic formulae (Dance and Maxey, 2003; Nguyen and Ladd, 2002) of lubrication force and torque can be applied to resolve the particle-wall interaction. For example, the normal force correction can be expressed as

$$F^{lub} = -6\pi \mu R u_{\perp}\left(\frac{1}{\varepsilon} - \frac{1}{\Delta}\right), \quad (1)$$

where $u_{\perp}$ is the particle velocity normal to the wall and $\Delta = \delta/R$ is a prescribed dimensionless distance cutoff below which the hydrodynamical force is compensated, δ is usually set to be one or two grid sizes, R is the radius of the particle, $\varepsilon \equiv h/R$ represents a small dimensionless gap, and h is the gap between the particle nose and the wall, μ is the dynamical viscosity coefficient of the fluid (ten Cate et al., 2002).

However, the current models for lubrication force and torque are based on the assumption of the Stokes flow, which might not be suitable for cases at finite particle Reynolds numbers. Here, the translational particle Reynolds number is defined as

$$Re_p = \frac{\rho_f u d_p}{\mu} \quad (2)$$

where $d_p = 2R$ is the diameter of particle, u is the particle velocity, and ρ_f is the density of fluid. Therefore, the objective of this work is to develop new models for the hydrodynamic force and torque at finite particle Reynolds numbers. The new models can be used in two major applications. First, they can be used as sub-grid models in PR-DNS calculations where the flow in the gap between the particle and wall can not be sufficiently resolved. They can be used in any PR-DNS approaches, such as lattice Boltzmann method (Ladd, 1994a; 1994b), immersed boundary method (Lucci et al., 2010) and fictitious particle method (Shao et al., 2012). Second, they can be used in Lagrangian simulations of particle-laden flows to replace the currently used model for an isolated particle. In current computational fluid dynamics (CFD) codes, force and torque models on the particles are not modified when they are very close to a wall. It will be shown in this work that the force and torque on a particle near a wall can be significantly different from those on an isolated particle. Therefore, the proposed new models will be useful for Lagrangian simulations of applications such as cyclone separators (Song et al., 2016).

In order to develop the models for hydrodynamic force and torque for particle motions in a fluid at low Reynolds numbers, superposition of four simple motions of two isolated spheres is usually used to analyze their lubrication forces and torques (Dance and Maxey, 2003; Rosa et al., 2011). The four simple motions consist of (i) a sphere translating normally to another sphere, (ii) two spheres translating along the direction perpendicular to the line connecting their centers, (iii) two spheres rotating around the line connecting their centers, (iv) two spheres rotating around the direction perpendicular to the line connecting their centers. For a sphere translating towards a stationary sphere, Jeffrey (1982) combined the work of Cooley and O'Neill (1969) with numerical calculations and deduced a brief formula proportional to ε^{-1} for normal force acting on the moving sphere. For the other three motions, Jeffrey and Onishi (1984) extended the work of O'Neill and Majumdar (1970) and obtained asymptotic expressions of the forces and torques acting on the spheres. Above all, the formulae were obtained based on the assumption of Stokes flow. The asymptotic formulae of the lubrication force and torque for the four simple motions are suitable for two spheres of different radii. When the radius of one of the two spheres tends to be infinite, the larger sphere could be regarded as a plane wall. Then the asymptotic formulae for two particles turned into the theoretical expressions of lubrication force and torque acting on a moving sphere near a plane wall (Dance and Maxey, 2003).

The theoretical expressions of lubrication force and torque are suitable for fluid flow at vanishing particle Reynolds numbers. However, there are many practical situations related to the motion of particles in fluid flow at finite particle Reynolds numbers. As for the particles in channel flows, Uhlmann (2008) fully resolved the phase interfaces by DNS with an immersed boundary method and García-Villalba et al. (2012) simulated turbulent flow in a vertical plane channel seeded with heavy spherical particles. Uhlmann and Doychev (2014) simulated the gravity-induced motion of finite-size particles in fluid in triply periodic domains. The particle Reynolds numbers of the flows are greater than 100. They used the repulsive force mechanism to recover the close particle-particle hydrodynamic interaction. Zeng et al. (2008; 2010) considered the turbulent channel flow over an isolated particle with variable sizes and locations. There are also pipe flows laden with particles, such as the experimental study of turbulent flow driven by particles in pipe flows (Belt et al., 2012) and the gas-solid flows in wall-bounded vertical risers (Laín and Sommerfeld, 2012; Lu et al., 2013; Wang et al., 2013). In addition, Lucci et al. (2010) numerically simulated the turbulent flow around moving spherical particles dispersed in a decaying isotropic turbulent flow. Yeo et al. (2010) investigated the modulation of isotropic turbulent flows induced by spherical bubbles, neutrally buoyant particles and slightly inertial particles. They also used the repulsive force between the particle surfaces when the gap is less than a critical value. In current large-eddy simulation of particle-laden channel flows, the unresolved sub-grid scale motion might affect the particle-wall interaction (Bianco et al., 2012). The contributions of the sub-grid scale fluid motions on particle-wall interaction can be partially modeled by constructing particle sub-grid scale model based on the space-time correction theory (He et al., 2002; Yang et al., 2008; Zhao and He, 2009).

The experimental investigations and numerical simulations described above are related to particle-particle and particle-wall hydrodynamical interaction at finite particle Reynolds numbers. The typical particle Reynolds numbers are respectively listed in Table 1.

Illustrated in Table 1 is the range of particle Reynolds numbers in many situations, the range is about $O(10) \sim O(100)$. Although the local particle Reynolds number near the wall can be reduced by the hydrodynamic force, the effects of finite particle Reynolds number will have to be considered. Experimental studies show that when the particle Stokes number $St = \tau_p/\tau_f = ((\rho_p/\rho_f)/9)Re_p$ is larger than a critical value, $St^* = 10$, the particle will approach the wall with a finite velocity and rebound back, where τ_p is particle relaxation timescale and τ_f is a characteristic timescale of the flow (Gondret et al., 2002; Joseph et al., 2001). Using the method of matched asymptotic expansions, Cox and Brenner (1967) considered the contribution of fluid inertia at a small but finite particle Reynolds number to the lubrication force by multiplying the Stokes drag force with a dimensionless friction factor $f_{zz}(\varepsilon)$,

$$f_{zz}(\varepsilon) = \frac{1}{\varepsilon} + \frac{1}{5}\left(1 + \frac{Re_p}{4}\right)\ln\left(\frac{1}{\varepsilon}\right) + O(Re_p^2), \quad (3)$$

where $\varepsilon \ll 1$ and $\varepsilon Re_p \ll 1$ and the particle approaches the wall with a constant speed. The moving conditions of a particle at constant translational or rotational speeds are also applied in this work. Liu and Prosperetti (2010) considered a sphere rotating at $Re_\Omega \le 200$ near one or two infinite plane walls parallel or perpendicular to the axis of rotation and studied the centrifugal, inertial and viscous effects on the hydrodynamic force and torque acting on the sphere. Here, the rotational particle Reynolds number is defined as

$$Re_\Omega = \frac{\rho_f \Omega R d_p}{\mu} \quad (4)$$

where Ω is the angular velocity of the sphere. Tagawa et al. (2013) investigated the wall effect on a repulsive force act-