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# An Eulerian model for the motion of granular material with a large Stokes number in fluid flow

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ABSTRACT

This study introduced a novel Euler–Euler approach for modeling granular multiphase flow. The motion of particles with a large Stokes number was investigated assuming that granular material has unilateral compressibility. Solid pressure in the momentum equations for granular multiphase flow was determined so that the unilateral incompressibility condition was satisfied. Using the continuity condition of the granular phase, the equation was rewritten in the optimal form to calculate the solid pressure. A discrete formulation of smoothed particle hydrodynamics was applied for the convective terms so that the discrete matrix was positive semidefinite for the convergence and the discretization for an unstructured mesh was allowed. Frictional stress was then determined from solid pressure and, by using the solid pressure and frictional stress, momentum equations for the granular phase were solved. The method was incorporated into ANSYS FLUENT by a UDF (user defined function). Model validation was performed through a comparison with two previous results, and efficacy of the proposed model was confirmed.



## 1. Introduction

Granular materials, such as sand, powders, cereal grains, and gravel, are commonplace in nature. Composed of large quantities of mesoscopic particles, such materials show unique physical behavior that is unlike other materials, including fluids and deformable bodies. In particular, granular materials disperse freely in free fall, flow plastically under force, and settle in stable piles.

For decades, the study of the behavior of these systems has interested many researchers. To date, the study of the motion of a particulate system has been categorized by two methods: Eulerian and Lagrangian. In the Eulerian method, granular particles are considered as a continuum, while in the Lagrangian method the motion of particles is determined individually.

In particular, the discrete element method (DEM), which was originally presented by Cundall and Strack, follows the Lagrangian approach and has been widely used to study granular flows (Cundall and Strack, 1979; Yamane et al., 1998; Zhu et al., 2008; Bluhm-Drenhaus et al., 2010; Zhou et al., 2011). This method considers translations or rotations of particles that come in contact with one another as well as those that do not. In DEM, Newton's momentum equations are solved for every particle to determine the motion of granular particles. These results can then be coupled with a set of equations for fluid flow to study granular multiphase flow.

This method is unaffected by the shape of particles because it accounts for six rotational degrees of freedom. However, additional calculations are needed to determine particle contact position (Kodaman et al., 2010; Tao et al., 2010). Additionally, information about the arrangement of every particle and whether or not each particle is in contact with a given particle is needed. Therefore, the detection speed of the contact between the particles is a key feature of DEM. In order to accelerate contact detection between particles using this method, many studies have been performed (Mio et al., 2007; Mio et al., 2009). Efforts to better describe this phenomenon have concentrated on improving the normal, tangential, and contact force models by considering the nonsphericity of the particles (Kodaman et al., 2010; Tao et al., 2010). Numerous examples for determining the motion of granular particles using DEM can be found, including feeding in the kilns and the furnaces (Zhu et al., 2008; Wright et al., 2011; Zhou et al., 2011). Although DEM can effectively describe granular particle motion by utilizing a Lagrangian approach and considering the interactions resulting from the collision between particles, it requires enormous computational effort.

In the Eulerian–Lagrangian approach, the Eulerian momentum equations are applied for fluid flow, while DEM is used for the granular phase. Then, the results are coupled through the inter-

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action between the overall flow and the individual particle to get a complete system solution (Drenhaus et al., 2010; Geng and Che, 2011).

As with smoothed particle hydrodynamics (SPH), there is another method in which the equations for fluid flow are solved using the Lagrangian approach (Li et al., 2007).

In the study of granular systems, the Eulerian–Eulerian method has also been used. Here, a granular system is considered to be a continuum where the particle motion is governed by Euler equations (Zhang et al., 2002; Modal et al., 2005; Demagh et al., 2012). Unlike DEM, in this approach it may be impossible to directly consider the shape of particles. In order to study a granular system with the Eulerian–Eulerian method, constitutive equations for the granular system are required. In other words, additional relationships for pressure and stress, which may occur due to collision and friction between particles, are needed.

Schaeffer (1987) and Johnson and Jackson (1987) derived constitutive relationships by applying the kinetic theory from phenomenological consideration. These relationships have been widely used in several practical problems, including granular flow in a fluidized bed (Zhang et al., 2002; Modal et al., 2005; Demagh et al., 2012).

Another technique to study granular flows is to combine the Eulerian–Eulerian approach with the Lagrangian method. Yuu and Umekage (2011) calculated the coefficients of the constitutive relationship using DEM and then applied the Euler–Euler approach to study the pile formation process.

Narain et al. (2011) assumed that granular material had unilateral incompressibility instead of a constitutive relationship and then reduced it to an optimization problem for solid pressure and stresses by discretizing the governing equations with the FLIP (Fluid-Implicit-Particle) method. They described the motion of the granular materials as a continuum equation and applied the Lagrangian numerical method.

The motivation of the current study is to present a new Eulerian–Eulerian approach for modeling granular multiphase flow with a large Stokes number and to incorporate it as a subroutine into commercially available CFD software, ANSYS FLUENT, via a user defined function (UDF). To do so, unilateral incompressibility was specified in ANSYS FLUENT instead of a constitutive relationship. The primary objective was to determine the solid pressure in the momentum equation for the granular phase by unilateral incompressibility.

Solid pressure due to friction between granular particles was formulated using an optimization problem obtained from the unilateral incompressibility hypothesis after discretizing a set of Euler multiphase equations for a solid phase. UDF code was written using this method and the results were compared with the experimental results for a rotational cylinder to determine the new method's validity. Finally, the results for various granular flows are provided.

## 2. Governing equation

It was assumed that both granular material and fluid were a continuum and had full inter-permeability. Therefore, their motion may be described by the continuity and momentum equations for a continuum.

### 2.1. Governing equations for fluid flow

Fluid flow in the multiphase medium with granular particles may be described as the following continuity and momentum equations:

$$\frac{\partial}{\partial t}\left(\varphi\rho_f\right) + \nabla\cdot\left(\varphi\rho_f\mathbf{v}\right) = 0 \tag{1}$$

$$\frac{\partial}{\partial t}\left(\varphi\rho_f\mathbf{v}\right) + \nabla\cdot\left(\varphi\rho_f\mathbf{v}\mathbf{v}\right) = -\varphi\nabla p + \nabla\cdot\varphi\tau + \varphi\rho_f\mathbf{g} + \mathbf{F}_{sf} \tag{2}$$

where $t$ is time, $\varphi$ is porosity, $\rho_f$ is fluid density, $p$ is fluid pressure, $\boldsymbol{\tau}$ is fluid viscous stress, $\mathbf{v}$ is the fluid velocity vector, and $\mathbf{F}_{sf}$ is the momentum source by interaction between the fluid and the particles.

Air flow turbulence is modeled by the standard $k-\varepsilon$ model:

$$\frac{\partial}{\partial t}\left(\varphi\rho_f k\right) + \nabla\cdot\left(\varphi\rho_f k\mathbf{v}\right) = \nabla\cdot[(\mu_f + \frac{\mu_t}{\sigma_k})\varphi\nabla k] + \varphi(G_k + G_b) - \varphi\rho_f\varepsilon - Y_M + S_k \tag{3}$$

and

$$\frac{\partial}{\partial t}\left(\varphi\rho_f\varepsilon\right) + \nabla\cdot\left(\varphi\rho_f\varepsilon\mathbf{v}\right) = \nabla\cdot[(\mu_f + \frac{\mu_t}{\sigma_\varepsilon})\varphi\nabla\varepsilon] + C_{1\varepsilon}\frac{\varphi\varepsilon}{k}(G_k + C_{3\varepsilon}G_b) - C_{2\varepsilon}\varphi\rho_f\frac{\varepsilon^2}{k} + S_\varepsilon \tag{4}$$

where $G_k$ represents the generation of turbulence kinetic energy due to the mean velocity gradients, $G_b$ is the generation of turbulence kinetic energy due to buoyancy, $Y_M$ represents the contribution of the fluctuating dilatation in compressible turbulence to the overall dissipation rate, $C_{1\varepsilon}$, $C_{2\varepsilon}$and $C_{3\varepsilon}$are constants. $\sigma_k$ and $\sigma_\varepsilon$ are the turbulent Prandtl numbers for $k$ and ε, respectively. $S_k$ and $S_\varepsilon$are the sources for $k$ and ε, respectively.

### 2.2. Governing equations for granular flow

The Eulerian multiphase equation for the motion of granular particles in the fluid flow can be written as follows:

$$\frac{\partial}{\partial t}(\varphi_s\rho_s\mathbf{v}_s) + \nabla\cdot(\varphi_s\rho_s\mathbf{v}_s\mathbf{v}_s) = -\varphi_s\nabla p - \nabla p_s + \nabla\cdot\tau_s + \varphi_s\rho_s\mathbf{g} + \mathbf{F}_{fs} \tag{5}$$

where $\varphi_s = 1-\varphi$ and $p_s$ is solid pressure, $\boldsymbol{\tau}_s$ is the stress formed due to the motion of granular particles, $\mathbf{F}_{fs} = -\mathbf{F}_{sf}$ is the source of momentum due to the interaction between the fluid and particles, $\rho_s$, $\mathbf{v}_s$ are the particle density and velocity vector, respectively.

The continuity equation for granular flow can be written as follows.

$$\frac{\partial}{\partial t}(\varphi_s\rho_s) + \nabla\cdot(\varphi_s\rho_s\mathbf{v}_s) = 0 \tag{6}$$

### 2.3. Modeling of solid pressure

To date, solid pressure has been described by granular temperature in multiphase flow theory (Modal et al., 2005; Demagh et al., 2012). This model was obtained from kinetic theory for very small particles (such as pulverized coal), and therefore is not adequate for particles with a high Stokes number. Stokes number is defined as the ratio of the particulate relaxation time to the relaxation time of the fluid flow, and is described as follows:

$$St = \frac{\tau_s}{\tau_f}$$

where $\tau_s = \frac{\rho_s d_s^2}{\mu_f}$, $\tau_f = \frac{L_f}{V_f}$, and $L_f$, $V_f$, $d_s$ are characteristic length, fluid system velocity, and particle diameter, respectively. When the Stokes number is large, particles are not tightly coupled with the fluid.

When a particle has a large diameter and material density much greater than that of the fluid, particle motion is not significantly affected by fluid flow. Therefore, it is not appropriate to describe granular pressure by granular temperature, which is a thermodynamic property. When the volume fraction of granular particles exceeds the packing limit, compressive pressure may be produced due to gravitational force. If granular materials are in motion