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Uncertainty quantification and global sensitivity analysis of mechanistic one-dimensional models and flow pattern transition boundaries predictions for two-phase pipe flows

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A B S T R A C T

The prediction of uncertainties is a growing interest in flow assurance industrial applications, but only few works have been presented on this topic. In this work, an uncertainty quantification and a global sensitivity analysis are performed to quantify the level of confidence in predictions of one-dimensional mechanistic models considering different two-phase flow regimes. A method is proposed for this purpose accounting for the effect of several variables on pressure drop and hold-up predictions by the well-known one-dimensional two-fluid model, such as fluid flow rates, geometry (the inclination angle and the pipe diameter), and fluid properties (density and viscosity); the case of a non-Newtonian shear-thinning fluid behaviour is also considered. Flow pattern transition boundaries, including the stability of the stratified flow regime, are included in this analysis. Monte Carlo simulations were used for the uncertainty quantification while different approaches for the sensitivity analysis (scatter plot, linear regression, the Morris's method, and the Sobol's Method) were used and compared to identify the best tool for this family of models. The Sobol's method appears to be the most convenient approach and a discussion is provided considering different practical cases for gas/liquid and liquid/liquid systems. The most critical input parameters in terms of uncertainty are rigorously identified case by case. A way to reduce the output uncertainty is indicated by the interpretation of the results of the global sensitivity analysis. The conclusions of this analysis gives new insights regarding the degree of uncertainties in predictions of one-dimensional mechanistic models.

1. Introduction

Multiphase flows in pipes are frequently encountered in many applications such as in chemical industry and during petroleum transport over long distances. An accurate prediction of flow characteristics, such as flow pattern, phase hold-up, and pressure drop, is a key aspect in the design process and during real time control strategies: mechanistic models, like the well-known one-dimensional two-fluid model (1D-TFM), are commonly used for this purpose. For given flow conditions, fluid properties, and pipe geometry these models predict steady state or transient flow characteristics in a large variety of flow conditions with an acceptable agreement, but, despite their success, only few studies have been published on the uncertainty quantification of these models. Even if the quantification of the level of confidence in predictions is a crucial challenge for the industry, a rigorous and systematic study on the estimation of uncertainties for this family of mechanistic models has not been proposed yet. In addition, due to the error in measuring the input data, an uncertainty quantification analysis is also needed for model validation.

Wilkens and Flach (2001) proposed to consider Taylor's series expansion (one-parameter-at-a-time approach) for the uncertainty propagation, but due to the high non-linearity of multiphase flow models and to the large uncertainty of the input parameters, other approaches are more suitable for this purpose. Posluszny et al. (2010) and Hoyer et al. (2013) performed Monte Carlo simulations to quantify the uncertainty on flow assurance systems, while Holm et al. (2011a, 2011b) implemented the Latin Hypercube Design (LHD) method. Pereyra et al. (2012) tested the confidence level of the predictions of the Barnea (1987) unified model for flow

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pattern transition on a database of nine thousands experimental points.

Only recently, Cremaschi et al. (2012) reviewed the existing approaches to express the degree of confidence in predictions for different multiphase flow mechanistic models, but the well-known 1D-TFM was not considered in that work. Soepyan et al. (2016) considered a Kriging Method for uncertainty propagation of solid particle transportation in pipes. On the other hand, the case of multiphase flow computational fluid dynamics simulations was investigated by Oberkampf et al. (2002), Gel et al. (2013), and Najm (2009), where more sophisticated techniques, such as of polynomial chaos, were considered (references are therein).

Despite one-dimensional two-fluid model and criteria for flow pattern transitions are widely used, these models were not included in previous studies of uncertainties quantification and propagation. In this paper, we present a structured and rigorous approach to quantify the level of confidence in predictions of the 1D-TFM model for different flow regimes. The purpose is not only to provide uncertainty bars of predictions, but to investigate how much the 1D-TFM amplifies the uncertainty of the input parameters. The idea is to identify the most dangerous input parameters in terms of uncertainty investigating which ones affects mainly the uncertainty of the predictions in order to reduce, when it is possible, the uncertainty of predictions.

Firstly, the uncertainty quantification is performed considering Monte Carlo simulations, while the sensitivity analysis is conducted considering different approaches, starting from scatter plots, linear regression, the one-parameter-at-a-time Morris method, and moving to a more sophisticated and mathematically rigorous method, the Sobol's method, for the first time for multiphase flow predictions. Due to the fact that in previous works the results were commented at most in terms of probability distributions of model outputs, we compared the results of the different approaches for the sensitivity analysis to identify the best tool for this family of models. Then, the Sobol's method has been proposed as the most convenient tool since Sobol's sensitivity indices accurately quantify the contribution of each input parameter (including the interaction effects) on uncertainty of predictions.

Our goal is to present a methodology and to test it on two-fluid model predictions referring to practical cases, distinguishing between gas/liquid and liquid/liquid systems: the most dangerous parameters are identified case by case computing the sensitivity indices to investigate in a rigorous way the degree of uncertainty of one-dimensional mechanistic models predictions. The conclusions of this analysis offer new insights on the relative importance of some of the input variables. In addition, the results of the Monte-Carlo Simulations and the Sobol's method can be used as a benchmark or a validation for more sophisticated uncertainty quantification method, like polynomial chaos method.

2. Theoretical considerations

In this section the methodology to perform the uncertainty quantification and the global sensitivity analysis on one dimensional mechanistic models for two-phase flows is presented.

2.1. One-dimensional mechanistic models for two-phase flows

The well-known one-dimensional mechanistic two-fluid models were not included in previous works on uncertainty quantification reviewed in the Introduction, thus, due to the relevance of this family of models for industrial applications, we focused on them distinguishing among different flow regimes:

- one dimensional two-fluid model for gas/liquid stratified flow, see Taitel and Dukler (1976), liquid/liquid stratified flow, see Brauner and Moalem Maron (1992), and gas/non-Newtonian liquid stratified flow, see Picchi et al. (2014) and Picchi and Poesio (2016b);
- one dimensional two-fluid model for liquid/liquid oil in water dispersed flow, see Picchi et al. (2015b);
- mechanistic model for gas/liquid slug flow regime, see Taitel and Barnea (1990), and extension to non-Newtonian fluids given by Picchi et al. (2015a);

All these models, which allow to predict pressure drop Δp and hold-up ε_b for steady state and fully developed conditions, are in the form

$$(\Delta p, \varepsilon_b) = f(Q_a, Q_b, D, \beta, \mu_a, \mu_b, \rho_a, \rho_b, m, n, L, \sigma_{ab}), \tag{1}$$

where the input variable $Q_{a,\,b}$, D, β, $\mu_{a,\,b}$, $\rho_{a,\,b}$, m, n, L, and σ_{ab} are the flow rate, the pipe diameter, the inclination angle (positive for downward flow), the Newtonian viscosity, the density, the non-Newtonian fluid consistency index, the non-Newtonian fluid behaviour index, the pipe length, and the surface tension, respectively. The subscripts a, b refers to the two phases, respectively.

Since the prediction of flow pattern transition is highly relevant, we considered also the criteria available for two-phase flows, such as the stratified flow stability boundaries, see Brauner and Moalem Maron (1992) for Newtonian fluids and Picchi et al. (2014) for non-Newtonian fluids; the results can be easily extended to the complete flow pattern map of Barnea (1987) and Picchi and Poesio (2016a). These criteria are in the form

$$(\text{boundary}) = f(Q_a, Q_b, D, \beta, \mu_a, \mu_b, \rho_a, \rho_b, m, n). \tag{2}$$

2.2. Uncertainty quantification (UQ): MonteCarlo method

Both the steady state predictions of the 1D-TFM and the criteria for flow pattern transitions described in Section 2.1 are in the form of a black-box type model, see for example Lee and Chen (2009),

$$y = f(x_1, .., x_i, ..., x_n), \tag{3}$$

where the input parameters x_i are n random scalar variables and y is the corresponding model output; in general, the input parameters can be correlated or dependent: since the absence of any information on such correlations, we will consider them as independent in this work. The function f includes also the numerical algorithm to calculate the predicted value and the uncertainty associated to each input parameter will be defined in Section 3.1.

Since these models have a cheap computational cost, we use a Monte Carlo based sampling technique for the uncertainty quantification (UQ). For a given model in the form of Eq. (3) the sampling $n \times N$ matrices $\mathbf{M}$ and the corresponding $N \times 1$ output-matrix $\mathbf{Y}$ can be generated (N is the sample number sufficiently large to ensure convergence). Then, the probability density function of the outputs and the mean $E(\mathbf{Y})$, the variance $V(\mathbf{Y})$, and the standard deviation σ (in case of normal pdf) are evaluated as

$$E(\mathbf{Y}) = \frac{1}{N}\sum_{j=1}^{N} Y_j, \quad V(\mathbf{Y}) = \frac{1}{N}\sum_{j=1}^{N}(Y_j - E(\mathbf{Y}))^2,$$
$$\sigma = \sqrt{\frac{1}{N}\sum_{j=1}^{N}(Y_j - E(\mathbf{Y}))^2} \tag{4}$$

2.2.1. Qualitative sensitivity analysis with scatter plots and regression analysis

Scatter plots give qualitative information about the input variables: this technique is cost-free, once the Monte-Carlo samples have been generated. If M_{ji} is the element of sampling matrix $\mathbf{M}$ (each row j corresponds to a Monte Carlo sample and each ith