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A Bayesian model selection analysis of equilibrium and nonequilibrium models for multiphase flow in porous media

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ABSTRACT

The classical constitutive relations for multiphase flows in porous media assume instantaneous and local phase-equilibrium. Several alternative nonequilibrium/dynamic constitutive relations have been proposed in the literature including the works of Barenblatt, and Hassanizadeh and Gray. This work applies a Bayesian model selection framework in order to examine the relative efficacy of these three models to represent experimental observations. Experimental observations of multiphase displacement processes in natural porous media are often sparse and indirect, leading to considerable uncertainty in control conditions. Data from three core-scale drainage experiments are considered. Gaussian prior probability models are assumed for key multiphase flow parameters and measurements. Accurate numerical simulation approximations using the three constitutive relation models are implemented. The model selection analysis comprises a data-assimilation stage that calibrates the assumed model to the data while quantifying uncertainty. The second stage is the computation of the maximum likelihood estimate and its application to compute the Bayesian Information Criterion. It is observed that Barenblatt's nonequilibrium model is more likely to match data from unstable displacements that involve higher viscosity ratios of the invading phase to the resident fluid. At the lowest viscosity ratio, there is no delineation between the goodness of fit obtained using the classical model and the model proposed by Hassanizadeh and Gray, and both outperform Barenblatt's nonequilibrium model.

1. Introduction

Macroscale models of multiphase flow in porous media rely on the multiphase extension of Darcy's law (Muskat and Meres, 1936), which is achieved via the inclusion of constitutive relations, i.e. relative permeability and capillary pressure functions. These are high-dimensional functions that depend on a number of parameters such as saturation history, initial wetting-phase saturation, wettability, interfacial tension, pore geometry, and viscosity of fluids and changes in any of the aforementioned as a result of changes in pressure and temperature (Avraam and Payatakes, 1999; Honarpour et al., 1986; Tang et al., 2004). The constitutive relations have been shown to not be unique functions of the instantaneous local saturation only, and to be closely related to the capillary number and the flow regime (Theodoropoulou et al., 2005; Tsakiroglou et al., 2003). In the classical formulation of multiphase flow in porous media, the complexity of the constitutive relations is largely ignored and the relations are assumed to be sole functions of local saturations (Aryana and Kovscek, 2012). One significant challenge of such formulations is their inability to adequately describe unstable multiphase flow processes (Riaz et al., 2007). Instabilities in flow dynamics are observed most prominently in the presence of high viscosity and/or density contrasts between invading and resident fluids (Aryana and Kovscek, 2012; Tang et al., 2004), where the classical assumption of instantaneous equilibrium of the local state of flow in porous media is no longer an adequate representation of the physics (Aryana and Kovscek, 2013; Barenblatt et al., 2003; Le Guen and Kovscek, 2006). To date, a number of nonequilibrium models have been proposed to address this deficiency. Examples of such models include the dynamic capillary pressure-saturation relation proposed by Hassanizadeh and Gray (H&G) (Hassanizadeh and Gray, 1993a; 1993b), Barenblatt's nonequilibrium (BNE) model (Barenblatt, 1971; Barenblatt and Gilman, 1987) and the general nonequilibrium formulation developed by Aryana and Kovscek (2013). Motivated by volume averaging of microscopic flow equations and thermodynamic considerations, Hassanizadeh and Gray define a dynamic capillary pressure that differs from the static

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capillary pressure by the product of a dynamic coefficient (τ) and the local rate of change of saturation (Hassanizadeh et al., 2002). Barenblatt argues that wetting and non-wetting phases, flowing simultaneously, establish separate networks of flow pathways that connect the flowing portion of each phase, and any change in capillary pressure causes a reconfiguration of flow pathways (Barenblatt et al., 2003). This results in changes in the connectivity of the flowing phases and the overall hydraulic conductance. It takes a finite amount of time to pass from one steady-state arrangement of flow pathways to another. The characteristic time of rearrangement of flow networks is referred to as redistribution (or relaxation) time. Alternate constitutive models include the extension of capillary pressure to dynamic conditions by Kalaydjian (1987), and dynamic capillary pressure relations that depend on specific interfacial areas (Joekar-Niasar et al., 2010; Niessner and Hassanizadeh, 2008). In this work, the classical, BNE and H&G models are selected for a Bayesian comparative analysis. The choice of BNE and H&G models is due to the fact that they both account for nonequilibrium effects by the inclusion of the product of the temporal rate of change of local saturation and a coefficient, which is referred to as relaxation or redistribution time in the case of BNE model, and dynamic coefficient in the case of H&G model.

In principle, the more parameters are included in a constitutive model, the better it tends to match available data. This does not, however, mean that the inclusion of a larger number of parameters produces a more predictive model. In this effort, a Bayesian inference framework is used to compare the complexity and bias of three candidate constitutive models (Aho et al., 2014) for macroscale multiphase flow through porous media. A model selection framework is applied to compare the relative efficacy of the classical, H&G and BNE models, using the Buckley–Leverett (Buckley and Leverett, 1942) type (BL-type) formulation, in terms of a balance between complexity and bias of the models. Experimental data comprising saturation interpretations of Computed Tomography (CT) scans of a strongly water-wet cylindrical Berea sandstone core undergoing a series of one-dimensional incompressible drainage displacements is considered. While a number of control parameters are certain, others are characterized by a prior probability distribution. H&G, BNE and the classical BL-type models are incorporated into accurate numerical simulation models of the experimental setup. By applying a data assimilation algorithm, the three models are calibrated to the series of experimental results, and for each case and model, a maximum likelihood match is obtained, given the prior model parameters. The Bayesian Information Criterion is then computed and compared.

2. Experimental setup and data

The centerpiece of the experimental setup is a strongly water-wet, cylindrical Berea Sandstone core, that is 5.08 cm in diameter and 60 cm in length (Aryana and Kovscek, 2012). A series of three displacement experiments are conducted. Prior to each experiment, the core is fully saturated with the wetting phase. In each experiment a nonwetting phase is injected at a rate of 1.41 cm^3/min, which translates to a superficial velocity of $U_t =$ 1 m/d. The particulars of the fluids used in each experiment are tabulated in Table 1.

The experimental setup is placed in the field of view of a Computed Tomography (CT) scanner, and the core is scanned at twelve preselected sections along the length of the core and at prescribed time intervals. In order to filter photons with lower energy from the beam and to help avoid beam-hardening, the core is positioned in the center of a tube made of precipitation-hardening aluminum alloy (6061) (Barrett and Keat, 2004). Raw CT data is collected and processed to estimate porosity and phase saturation distributions (Aryana and Kovscek, 2012). According to Beer's law, the porosity for each voxel is determined as:

Table 1
Experimental fluids and capillary numbers.

Exp #	Invading	Resident	vr[c]	N_{ca}[d]
1	n-decane	brine[a]	1.1	2×10^{-7}
2	n-pentane	brine[a]	4.4	4×10^{-7}
3	n-decane	mix15[b]	15.0	1×10^{-6}

[a] 8% (by weight) sodium bromide solution.
[b] mixture of 8% (by weight) sodium bromide solution and glycerin at a volume ratio of 5:8.
[c] viscosity ratio of the resident to the invading phase.
[d] capillary number.

$$\phi = \frac{CT_{wet} - CT_{dry}}{CT_w - CT_a}, \tag{1}$$

where CT is proportional to attenuation of X-ray due to its passage through the setup and has units of Hounsfield (H). CT_{wet} and CT_{dry} refer to CT numbers associated with fully water and air saturated rock, respectively. CT_w and CT_a represent CT numbers associated with pure water and air, respectively. The non-wetting phase saturation for each voxel is calculated as,

$$S_n = \frac{CT_{exp} - CT_{wet}}{\phi\ \left(CT_n - CT_w\right)}, \tag{2}$$

where CT_{exp} are CT numbers associated with the rock during experiments, CT_{wet} are CT numbers obtained while the rock is fully saturated with wetting phase, ϕ is the voxel by voxel matrix of porosity at the particular image location, and CT_w and CT_n are CT numbers of pure wetting and nonwetting phases, respectively. Eqs. (1) and (2) yield porosity and saturation values for voxels that make up the scanned sections of the core.

Mercury porosimetry is used to establish pore size distribution and capillary pressure for the sandstone core. The capillary pressure data, shown in Fig. 1a, is corrected to represent the pairs of fluids used in experiments in accordance with the approximation of the Young-Laplace equation,

$$p_{c_{ow}} = \frac{\gamma_{ri} cos\theta_{ri}}{\gamma_{ma} cos\theta_{ma}} \approx \frac{\gamma_{ri}}{\gamma_{ma}}, \tag{3}$$

where γ is interfacial tension, θ is wetting angle, and subscripts *ri* and *ma* refer to resident-invading and mercury-air pairs of fluids, respectively. The Lucas–Washburn equation is used to calculate pore diameters. The majority of the pores fall within a narrow range of about 9 μm–30 μm in size. The concentration of pore-size distribution of the core in such a narrow range, along with its relatively constant values of measured porosity provide evidence for its homogeneity. This is particularly evident within the cross-sections normal to flow where little variation in porosity is detected. The average porosity at each scan location is shown in Fig. 3. The distribution of porosity in the core, expressed in terms of the frequency of deviations of porosity values measured at each voxel from the average porosity for the entire core, is shown in Fig. 2. Consistent with these observations, it is assumed that the porosity field throughout the core is homogeneous with a Gaussian uncertainty model (mean = 0.189 and standard deviation = 0.01). In line with the relatively linear pressure drop along the core during displacement experiments, and the proportionality of permeability to porosity and the assumption of homogeneity, it is also assumed that the permeability field is homogeneous. Data from single phase (brine) displacement experiments under constant flow rate conditions was used to infer that the effective permeability in the core is 0.25 D μm^2.