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Collision frequency and radial distribution function in particle-laden turbulent channel flow

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ABSTRACT

We performed Eulerian–Lagrangian direct numerical simulation of particle-laden channel flow at a frictional Reynolds number of 950. A fully parallelized deterministic particle collision algorithm is applied for elastic collisions between two particles and particles and the walls. A total number of 51 million mono-disperse particles is considered, resulting in a particle volume fraction close to 1×10^{-4}. We studied the results of the simulation after a statistically steady turbulent state and particle concentration were reached. In this state the particles close to the walls are preferentially located in the low-speed streaks, whereas the particle distribution in the center of the channel is rather non-uniform as well, showing large void regions. The presence of the particles results in a decrease of the turbulence dissipation rate of 20% close to the walls. We studied in particular the particle collision frequency as a function of the wall-normal coordinate and compared the simulation results with two theoretical expressions involving the radial distribution function at contact and the mean relative velocity of two colliding particles. It appeared that the radial distribution function diverges if the distance between the particles approaches the particle diameter. This is not only caused by the drift mechanisms but also by repeated collision events, which occur relatively more often in the center of the channel. We proposed a collision criterion that distinguishes repeated collisions from single collisions, and which is easy to apply during a simulation and in the computation of the radial distribution function and the mean relative velocity of two colliding particles. Simulation results and the two theoretical expressions for the collision frequency are in good agreement if this criterion is applied.

1. Introduction

The collision process of droplets or solid particles in turbulent flows is relevant in many flows in nature and technological applications. A crucial quantity is the collision frequency f, the number of collisions per unit time and volume. Let us consider a monodispersed system of spherical particles. We denote the particle diameter by d_p, the number of particles per unit volume by n, and the relative velocity between two particles just before collision by $\mathbf{w}$. Sundaram and Collins (1997) derived the following estimate for the collision frequency for inertial particles in a turbulent flow:

$$f^{(1)} = \frac{1}{2} n^2 \pi d_p^2 \langle |\mathbf{w}| \rangle g(d_p). \quad (1)$$

The angular brackets denote the statistical mean, and $g(d_p)$ denotes the particle radial distribution function at contact. For a statistically uniform spatial distribution of particles $g(d_p) = 1$, and then the estimate reduces to the one known from classical kinetic theory, based on the notion that the mean cylindrical volume swept by a single particle per unit time is equal to $\pi d_p^2 \langle |\mathbf{w}| \rangle$. Thus, Eq. (1) is called the cylindrical estimate of the collision frequency.

A second estimate of the collision frequency is the so-called spherical formulation (Wang et al., 2000),

$$f^{(2)} = n^2 \pi d_p^2 \langle |w_r| \rangle g(d_p), \quad (2)$$

in which w_r denotes the radial component of the relative velocity just before collision. According to Saffman and Turner (1956), the collision frequency in turbulent flows can be based on the net inward flux into a sphere of radius d_p around a particle, and this flux can be estimated by half the area of this sphere multiplied with the mean of the radial component of the relative velocity: $2\pi d_p^2 \langle |w_r| \rangle$. The original expression of Saffman and Turner (1956) for the collision frequency of small non-inertial particles in isotropic turbulence can be derived from Eq. (2) with $g(d_p) = 1$.

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Zaichik et al. (2003) presented two models for the particle collision frequency for homogeneous isotropic turbulence valid for the whole range of particle inertia from passive particles to the ballistic limit based on the spherical formulation. One of the models is based on the assumption that the velocities of particle and fluid obey a correlated Gaussian distribution, whereas the other starts from a kinetic equation for the probability distribution function of the relative velocity of two particles. Both models give adequate results in comparison to DNS results of particle-laden homogeneous isotropic turbulence. In a later paper, this work was extended to bi-disperse particles (Zaichik et al., 2009).

Concepts and arguments as those described above, in particular the cylindrical concept, are typically used to arrive at stochastic models suitable for practical applications in, for example, spray technology. With the ultimate aim to improve such models and also to obtain more insight into the fundamentals of the collision process, it is interesting to investigate whether the expressions above are suitable descriptions for the actual collision frequency in particle-laden turbulent flows. This question has been investigated in the past, but only for isotropic turbulence, see Salazar and Collins (2012), Salazar et al. (2008), Sundaram and Collins (1997), Wang et al. (2000). Reade and Collins (2000), Wang et al. (1998, 2000) concluded that the second expression is more accurate than the first expression, at least for homogeneous isotropic turbulence and for a case in which the actual contact frequencies were monitored but the collisions themselves were not performed (the particles could move through each other). In practical applications, turbulence is often inhomogeneous and anisotropic, while particles cannot move through each other.

The purpose of this paper is therefore to investigate the particle collision process and collision statistics (in particular statistics related to the stochastic estimates of the collision frequency above) for an inhomogeneous particle-laden turbulent flow at reasonably high Reynolds number, taking into account all collisions. More specifically, we perform a point-particle direct numerical simulation of a turbulent channel flow at friction Reynolds number Re_τ equal to 950, including 51 million small inertial particles. We compute and investigate the collision frequency, the particle radial distribution function and the components of the particle relative velocity vector $\mathbf{w}$ at various distances from the wall. The contents of this paper is as follows. The governing equations and numerical algorithm are specified in Section 2. The results of the simulation are discussed in Sections 3 and 4 contains a discussion and a summary of the conclusions.

2. Governing equations and numerical methods

The system consists of a continuous phase, the gas, which is described in an Eulerian way, and a dispersed phase, discrete particles, which are described in a Lagrangian way. The two phases are coupled by the drag force exerted by the gas on the particles and the resulting feedback force on the gas. Moreover, particles interact by collisions. The models and numerical methods for the two phases and the particle collisions are described in the following subsections. We consider fully-developed turbulent channel flow in the absence of gravity with periodic conditions in both streamwise and spanwise direction. Throughout the paper we will use x as the streamwise coordinate, y as the wall-normal coordinate and z as the spanwise coordinate. The size of the computational domain equals $2\pi H$ in streamwise direction and πH in spanwise direction, where H is half the channel height.

2.1. Continuous phase

The continuous phase is treated in an Eulerian way and assumed to be incompressible. The volume fraction of the particles is taken so small that the only effect of the particles on the gas is the two-way coupling force. Therefore, the gas satisfies the continuity equation for incompressible flow,

$$\nabla \cdot \mathbf{u} = 0, \tag{3}$$

where $\mathbf{u}$ is the velocity of the gas. Moreover, the gas momentum equation is modeled by the Navier–Stokes equation for incompressible flow, supplemented with a model for the interaction force between the two phases:

$$\frac{\partial \mathbf{u}}{\partial t} + \omega \times \mathbf{u} + \nabla P = \nu \Delta \mathbf{u} + \frac{\mathbf{b}}{\rho_g} + \frac{\mathbf{f}_{2w}}{\rho_g}, \tag{4}$$

where $\omega = \nabla \times \mathbf{u}$ is the vorticity, $P = p/\rho_g + \frac{1}{2}\mathbf{u}^2$, ν and ρ_g are the kinematic viscosity and mass density of the gas, p is the static pressure, and $\mathbf{b}$ is the driving force density necessary to maintain the flow. Finally, $\mathbf{f}_{2w}$, where the subscript 2w stands for two-way coupling, describes the momentum exchange between the two phases that will be specified in Section 2.3. The driving force density is taken constant in space and time and is directed in the streamwise direction.

In the two periodic directions a Fourier–Galerkin approach is chosen, whereas the wall-normal direction is treated by a Chebyshev-tau method. The incompressibility constraint is satisfied by using the wall-normal component of the vorticity vector and the Laplacian of the wall-normal velocity component as dependent variables, instead of the three velocity components. Hence, the spatial discretization of the problem for the gas velocity closely follows the method by Kim et al. (1987). Nonlinear terms are calculated in physical space by fast Fourier transform (FFT) with application of the 3/2 rule in both periodic directions. For integration in time a combination of a second-order accurate three-stage Runge–Kutta method and the implicit Crank–Nicolson method is chosen according to Spalart et al. (1991). In this way the nonlinear terms are treated in an explicit way, whereas the linear terms are treated implicitly. This method has been used and validated extensively at various frictional Reynolds numbers ranging between 180 and 950 (Kuerten and Brouwers, 2013; Vreman and Kuerten, 2014a,b).

The magnitude of the driving force results in a frictional Reynolds number equal to $\mathrm{Re}_\tau = 950$, which corresponds to a bulk Reynolds number of approximately 19,000, in the absence of two-way coupling between particles and gas. The mass fraction of particles is so low that the effect of two-way coupling on the frictional Reynolds number is less than 1%. The numbers of Fourier modes in the two periodic directions equal 768, whereas 385 Chebyshev polynomials are used in the wall-normal direction. The resolution in the wall-normal direction is equal to the one applied by Hoyas and Jiménez (2008). The grid spacing equals $7.8\nu u_\tau^{-1}$ in streamwise direction and $3.9\nu u_\tau^{-1}$ in spanwise direction, where u_τ is the friction velocity. This is a higher resolution than used by Hoyas and Jiménez, but our computational domain is much smaller. The time step used in the simulation equals $0.095\nu u_\tau^{-2}$. Results for single-phase flow agree well with the results by Hoyas and Jiménez (2008).

2.2. Dispersed phase

The dispersed phase consists of solid, spherical particles, which are treated in a Lagrangian way, by solving equations for the position and velocity of each individual particle. Furthermore, the particles are sufficiently small to allow a point particle approach. Since the mass density of a particle is several orders of magnitude larger than the mass density of the gas, the only relevant force between the two phases is the drag force (Armenio and Fiorotto, 2001). Moreover, we do not take gravity into account. For lower Reynolds number, a comparison between a case with and without gravity is shown in Kuerten and Vreman (2015), revealing that