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Inertial effects on the flow of capsules in cylindrical channels

Robert M. Carroll, Nivedita R. Gupta*

Department of Chemical Engineering, University of New Hampshire, W313 Kingsbury Hall, 33 Academic Way, Durham, NH 03824, United States

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ABSTRACT

A front-tracking method was used to study moderate to large-sized capsules flowing in cylindrical channels at Reynolds numbers ranging from 0.1 to 225. Two different constitutive equations, the Neo-Hookean and Skalak laws were considered to describe the mechanics of the thin membrane. The effect of capsule size, elastic capillary number, and Reynolds number on the shape, migration velocity, and extra pressure loss were determined. The deformation of the capsules was strongly tied to the size of the capsule compared with the channel diameter with larger capsules deforming more due to the confining effect of the wall. As the Reynolds number was increased, capsules were more elongated in the direction of flow. The effect of Reynolds number was more apparent as the elastic capillary number was increased. Both the migration velocity and extra pressure loss were seen to depend primarily on the size of the capsule with deformation playing a secondary role. The Neo-Hookean membrane showed a larger deformation than the Skalak law due to its strain softening nature. The Neo-Hookean membrane also displayed a failure phenomenon of continuous deformation at large enough elastic capillary numbers not seen in the Skalak law membranes. This limiting elastic capillary number was shown to decrease as Reynolds number became larger. The membrane strain was largest at the front of the capsule indicating the most likely region where the capsule would fail.

1. Introduction

Capsules consist of liquids or gels enclosed by a thin walled elastic membrane typically used to protect sensitive chemicals such as drugs or flavoring compounds for controlled release (Duncanson et al., 2012; Nazzaro et al., 2012; Zhang et al., 2013b). During capsule formation and use it is important to keep the membrane deformation below a critical level or it can rupture and prematurely release its contents (Chang and Olbricht, 1993b; Li et al., 1988). The level of strain within the membrane can be predicted using a continuum model in which the capsule consists of a liquid drop surrounded by a two dimensional elastic membrane (Pozrikidis, 2003). Human red blood cells (RBC), in particular have received a lot of attention as they have a relatively simple non-nucleated interior which can be modeled as a Newtonian liquid (Fung and Zweifach, 1971; Skalak et al., 1973). The flow and deformation of RBCs in small capillaries has been studied by a variety of authors to better understand human microcirculation (Secomb, 1993; Secomb and Hsu, 1996; Pozrikidis, 2005b; Chien et al., 1971).

As capsules are often formed and processed in liquids it is of interest to study the flow induced deformation of capsules. Barthès-Biesel (1980) provided a small deformation analysis for the shape of a thin walled capsule in general linear flows. This work has been used to estimate elastic properties of artificial capsules based on their deformation by Chang and Olbricht (1993a) and by Walter et al. (2000). The analytical solutions are limited to small deformation studies in unbounded domains. For larger deformations, there have been many numerical simulations of capsules in shear flow which have elucidated the effects of different membrane constitutive equations (Barthes-Biesel, Diaz, and Dhenin, 2002), membrane bending rigidity (Pozrikidis, 2001a), and the unstressed shape of the capsule (Bagchi and Kalluri, 2009). The continuum model of a capsule has also been used to understand the mechanical properties of biological cells (Lim et al., 2006; Evans and Skalak, 1980). In addition to shearing flows, a lot of attention has been given to the flow of capsules in channels. Early research on channel flow was conducted by Lee and Fung (1969) who used thin walled rubber capsules flowing in cylindrical channels as model for blood flow. Leyrat-Maurin and Barthès-Biesel (1994) used a boundary integral technique to study the motion of a capsule through a channel with a constriction. They showed that as the capsule moved through the narrowest portion of the channel it significantly increased the flow resistance and could sometimes plug the channel. Numerical simulations of capsule flow have been performed in cylindrical channels (Lefebvre and Barthes-Biesel, 2007; Pozrikidis, 2005a; Queguiner and Barthes-Biesel, 1997), and square channels (Hu et al., 2012; Kuriakose and Dimitrakopoulos, 2011; Doddi and Bagchi, 2009) by

* Corresponding author.
E-mail address: ngupta@unh.edu (N.R. Gupta).

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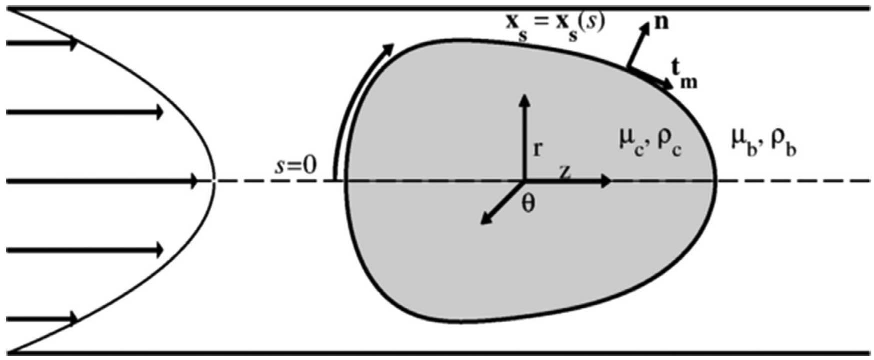

Fig. 1. Description of capsule geometry and surface vectors. The flow is from the left to the right with the arc length starting from the rear of the capsule where $s = 0$.

several authors in the Stokes flow limit. These studies have elucidated the effects of varying the elastic properties of the membrane and the size and shape of the capsules. Hu et al., (2012) performed fully three dimensional studies of capsule flow in cylindrical and square channels and showed that the capsules are less deformed in square channels at similar flow conditions. The predictions of capsule shapes in the numerical simulations have been used to calculate material properties of artificial capsules in channel flow (Chu et al., 2013; Lefebvre et al., 2008; Dimitrakopoulos and Kuriakose, 2015).

In the current paper we examine the flow of capsules in cylindrical channels when inertial effects are important, such as flow through ventricular assist devices (Fraser et al., 2011) or through syringes (Aguado et al., 2011). Microfluidic devices have been created that rely on inertial forces to sort cells emphasizing the need for a better understanding on the role of inertia in these flows (Mach and Di Carlo, 2010; Zhang et al., 2013a). To date most of the work on inertial effects on capsule deformation has been limited to the case of capsules in shear flow (Bai et al., 2013; Salac and Miksis, 2012; Sui et al., 2008; Doddi and Bagchi, 2008) with a few studies on capsule flow in channels (Kilimnik et al., 2011; Shin and Sung, 2012). Bai et al. (2013) showed that the deformation and the time scale for deformation increased with the Reynolds number. Doddi and Bagchi (2008) made a phase diagram of the different pairwise interactions seen between capsules in shear flow at finite *Re*. The few numerical studies on inertial effects in channel flow have looked at lateral migrations (Kilimnik et al., 2011) and tumbling motions of oblate capsules (Shin and Sung, 2012). Kilimnik et al. (2011) showed that the capsules deformation increased and the equilibrium position changed with the Reynolds number. However, to date there have been no studies that examine the effects of inertia on the deformation of confined capsules in channels.

In the current study we focus on the inertial effects of an axisymmetric capsule flowing in Poiseuille flow in a cylindrical channel. A front-tracking method, previously used to study droplet flow in channels (Carroll and Gupta, 2014), was modified to include an elastic membrane that provides a more localized treatment of the elastic tension and bending moments along the membrane. The numerical method is first validated against the test case of a capsule in an extensional flow field. Simulations were performed with channel Reynolds number ranging from 0.1 to 225. We examine the deformation of capsules with increases in *Re* and relate the deformation to changes in the capsules velocity and extra pressure loss.

2. Problem formulation

We consider a single, thin walled, initially spherical, capsule flowing through a cylindrical channel as seen Fig. 1. The capsule is taken to be neutrally buoyant with the density of the fluid inside ρ_c and outside of the capsule ρ_b, set equal and the mass of the capsule membrane considered to be negligible. The non-dimensional capsule size $\kappa = a/R$, where a is the undeformed capsule radius and R is the channel radius, is taken to be large enough that hydrodynamic forces keep the capsule on the channel centerline. The flow is therefore axially symmetric and we only consider the motion in the *r*-*z* plane. Fluid enters the channel at a constant flow rate Q, giving an average velocity $U = Q/(\pi R^2)$. The suspending fluid and the capsule's internal fluid are taken to be incompressible and Newtonian with equal viscosities, $\mu_c = \mu_b$. The location of the capsule membrane separating the two fluids is denoted by x_s.

The capsule is placed at the center of a long channel of length L, where the inlet and outlet boundaries are sufficiently far from the capsule that the flow is undisturbed. As the Reynolds number increases the flow disturbances from the capsule take longer to decay away requiring channels up to $L = 20R$. The capsule is set in a non-inertial reference frame $\mathbf{u}_{\text{ref}}$, which tracks the center of mass of the capsule. As we are interested in inertial effects on the capsule motion we solve the full Navier–Stokes equations written in conservative form as

$$Re\left[\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{uu})\right] = -\nabla P + \nabla \cdot \boldsymbol{\tau} - Re\frac{\partial \mathbf{u}_{\text{ref}}}{\partial t}, \quad (1)$$

$$\nabla \cdot \mathbf{u} = 0, \quad (2)$$

with the viscous stress tensor given as $\boldsymbol{\tau} = [(\nabla \mathbf{u}) + (\nabla \mathbf{u})^T]$ and the channel Reynolds number based on the bulk fluid properties $Re = \rho_b UR/u_b$.

The no slip condition is imposed on the rigid walls of the tube and the inlet flow is specified to be parabolic, Poiseuille flow. The velocity field is assumed to be continuous at the interface and the membrane moves and deforms with the local velocity

$$\frac{d\mathbf{x_s}}{dt} = \mathbf{u}(\mathbf{x_s}). \quad (3)$$

Because we have assumed the interface to be massless the difference in stress of the two fluids exerts a load on the interface $\mathbf{f}$,

$$\mathbf{f} = [\boldsymbol{\Pi}_c(\mathbf{x}_s) - \boldsymbol{\Pi}_b(\mathbf{x}_s)] \cdot \mathbf{n}, \quad (4)$$

where $\boldsymbol{\Pi}_c$ and $\boldsymbol{\Pi}_b$ are the total stress tensors in the capsule and bulk fluids, and $\mathbf{n}$ is the outward pointing normal vector. Force equilibrium within the membrane requires a balance between the in-plane elastic tensions $\mathbf{T}$, and transverse shear tension $\mathbf{q}$, with the external load $\mathbf{f}$ as

$$\mathbf{f} = \frac{1}{Ea}\nabla_s \cdot (\mathbf{T} + \mathbf{qn}), \quad (5)$$

where $\nabla_s = (\mathbf{I} - \mathbf{nn}) \cdot \nabla$ is the surface gradient operator and $Ea = \mu_b U/G_s$ is the elastic capillary number based on the shear modulus