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Semi-analytical solutions for the transient temperature fields induced by a moving heat source in an orthogonal domain

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ABSTRACT

A semi-analytical solution has been derived for the transient temperature fields that are generated in a three-dimensional solid body when it is subjected to one or more moving heat sources. The solution was derived using the Green's function method, and is applicable to any orthogonal domain that is subject to arbitrary boundary conditions. The solution can account for any linear combination of double-ellipsoidal or double-ellipsoidal-conical (DEC) heat sources. It can therefore be applied in situations ranging from an electric arc moving across a flat plate, to partial-penetration or full-penetration welding either with a laser or an electron beam. In this work, full penetration electron beam welds in 30 mm and 130 mm thick sections of SA508 steel were used as experimental test cases. The solution was shown to offer improved accuracy and dramatic reductions in solution times when compared with numerical methods, thereby lending itself to real-time in process monitoring of fusion welding processes.

1. Introduction

The simulation of physical processes in which changes of state occur, with a view to generating predictions for transient temperature fields, is computationally expensive. Advances have been made using fully coupled thermal-fluid simulations using source terms in the Navier-Stokes equations in order to arrest the motion in the solid state [1,2], and source terms in the energy equations to account for latent heat and energy generation [3]. In the modelling of fusion welding processes, another option for the prediction of transient temperature fields is to neglect mass transport associated with weld pool convection and other phenomena, and to solve the heat equation with a representative model of the welding heat source. Three-dimensional volumetric heat sources have been shown to produce the most accurate predictions in such scenarios [4–7]. Nevertheless, both the thermal-fluid solution procedure, and the heat equation solution procedure, are subject to spatial discretisation errors and both rely on closely spaced calculation points in order to predict the evolution of the various fields, particularly the temperature field [8]. Alternatively, semi-analytical techniques may be used [9].

Semi-analytical approaches involve determining the thermal response of a system to an infinitesimally small impulse of heat, and integrating this response over the domain, which is perturbed by the chosen heat source distribution [10,11]. This methodology again neglects any motion in the liquid state and produces a solution to the heat equation. Heat kernels are calculated analytically and then integrated numerically over a desired time interval, which is why such approaches are often referred to as semi-analytical techniques. The most mature of these approaches is the Greens function method [11,12].

A disadvantage associated with semi-analytical methods is that the domain over which the heat kernels are calculated must be orthogonal in order to construct appropriate Dirichlet and/or Neumann boundary conditions using the method of images (MOI). This often limits the application of these heat kernels to idealised scenarios [9]. However, certain physical processes do involve the application of a distributed source of heat to an orthogonal domain, with one example being the electron beam (EB) welding process when applied to the butt-welding of flat plates. Indeed, any fusion welding process, whether it involve an electric arc, a laser or an electron beam, will satisfy these conditions provided that a square-butt weld configuration is employed, regardless of whether the

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resulting weld fully penetrates the plates or not. In such cases, the calculation procedure for the heat kernels fundamentally maintains that they are free from discretisation errors, and this is a major advantage of the semi-analytical solution procedure over schemes in which the spatial thermal response is calculated over a discretised grid [13]. Clearly, for either numerical or semi-analytical solution procedures to be representative of a process, an appropriate heat source model must be used [4].

In this work, we present a semi-analytical solution for a welding heat source travelling across an orthogonal domain, and we validate the solution using experimental data that was obtained from the EB welding process. This process utilises a focused beam of high velocity electrons to generate heat at the interface between two mating surfaces. The material in the vicinity of the seam melts and flows together [3]. The high power density causes localised vaporisation, and vapour pressure leads to the formation of a capillary region in the molten weld pool that is often called a keyhole [14]. A keyhole will not form if the power density is insufficient to cause significant vaporisation and, as such, the penetration depth of an incident electron beam into the parent material is related to the beam power density [15]. The prediction of transient temperature fields for the electron beam welding process is of significant interest to the nuclear industry, where models for micro-structural evolution and the development of residual stresses rely on the accurate prediction of welding thermal cycles.

Recently, a heat source model was developed that can represent welding processes in scenarios ranging from an electric arc impinging on a flat plate to an electron beam being applied in the keyhole mode. This model is referred to as the double-ellipsoidal-conical (DEC) heat source model [7]. In this work, the DEC heat source model is incorporated into a Greens function semi-analytical solution procedure and used to predict the transient temperature fields for two EB welding scenarios. Given the inherent flexibility of the DEC heat source model, the solution presented in this work will have numerous applications, and these will not be limited to EB welding. The solution will be applicable to any situation in which an orthogonal domain is subjected to a heat source, or any linear combination of heat sources, where the DEC model can represent each heat source. Furthermore, the computational efficiency associated with this semi-analytical solution may generate opportunities for the application of this approach in real-time process monitoring for high energy density welding processes.

2. Semi-analytical solution procedure

For transient heat conduction, a Green's function can describe the temperature distribution caused by an instantaneous, local heat pulse. The Green's function for a given geometry and set of homogeneous boundary conditions is a building block for the temperature distribution due to a functional initial temperature distribution, and an internal heat flux distribution, q [16]. Multiplication of one-dimensional Green's functions may be used to construct solutions of higher dimensional order [10]. In order to compute the temperature field produced by the DEC heat source model, one must calculate the thermal response due to an infinitesimally small heat quantity, δ, acting instantaneously at time t' and at point (x', y', z') in an infinite domain [9,10]. This response may then be summed with the thermal responses associated with all equivalent heat pulses acting throughout the volume of the domain.

If δ acts instantaneously at time t' and at point (x', y', z') in an infinite domain the infinitesimal rise in temperature due to this point heat pulse $dT_{(x,y,z,t)}$ is given by $\frac{\delta}{\rho c_p} K_{(x,x',y,y',z,z',t,t')} dt'$, where K is the three dimensional fundamental solution for an infinite domain, as given by Equation (1) [17]. Here, ρ is the mass density and c_p is the specific heat at constant pressure, $\alpha = k/_{\rho c_p}$ is the thermal diffusivity, and k is the thermal conductivity. The temperature increase at a point (x, y, z, t) is then found by integrating from 0 to t.

$$K_{(x,x',y,y',z,z',t,t')} = \left\{ \begin{matrix} \dfrac{1}{(4\pi\alpha(t-t'))^{3/2}} e^{-\frac{(x-x')^2+(y-y')^2+(z-z')^2}{4\alpha(t-t')}} \quad \forall\ (t-t') \geq 0 \\ 0 \quad \forall\ (t-t') < 0 \end{matrix} \right\} \tag{1}$$

The MOI may then be used to adapt the solution from one over an infinite domain to one over a finite domain with insulating and Dirichlet boundary conditions, where $\frac{\partial T}{\partial x}$ and T are assigned, respectively, by placing equivalent fictitious sources to absorb or reflect the heat.

Consider an orthogonal domain with dimensions of B, D and L in the x, y and z directions, respectively. Cole et al. [11] demonstrated that the 1D Green's function for insulating boundary conditions ($\partial G/\partial x = 0$, at $x = 0$ and $x = B$) is given by Equation (2), and the 1D Green's function for $\partial G/\partial x = 0$ at $x = 0$ and a Dirichlet boundary condition at $x = B$ is given by Equation (3).

$$G_{(x,x',t,t')} = \sum_{n=-\infty}^{\infty} \frac{e^{-\frac{((2nB)+x-x')^2}{4\alpha(t-t')}} + e^{-\frac{((2nB)+x+x')^2}{4\alpha(t-t')}}}{\sqrt{4\pi\alpha(t-t')}} \tag{2}$$

$$G_{(x,x',t,t')} = \sum_{n=-\infty}^{\infty} (-1^n) \frac{e^{-\frac{((2nB)+x-x')^2}{4\alpha(t-t')}} + e^{-\frac{((2nB)+x+x')^2}{4\alpha(t-t')}}}{\sqrt{4\pi\alpha(t-t')}} \tag{3}$$

For a three dimensional domain with insulating boundary conditions on all faces except a Dirichlet boundary condition on the $x = B$ face the three dimensional Green's function may be found by multiplying the appropriate 1D functions and is shown in Equation (4).