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Mixed convection in a vertical circular microchannel

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ABSTRACT

The paper focuses on a study of mixed convection in a vertically oriented circular microchannel with slip boundary conditions. Novelty of the work consists in analytical and numerical solutions of the problem using the Lattice Boltzmann method (LBM). The novel analytical solution enabled obtaining relations for velocity and temperature profiles and Nusselt numbers depending on the Knudsen, Rayleigh and Prandtl numbers. Knudsen number effects dominate in the vicinity of the channel wall, whereas close to the channel axis the Rayleigh number effects prevail. For high Rayleigh numbers, velocity profiles transform themselves to M-shapes with a minimum point at the channel axis, and temperature profiles flatten out and tend to the wall temperature. The magnitude of the temperature jump on the channel wall is a function of the Prandtl number and diminishes for high Prandtl numbers. For almost all computed cases, larger Knudsen numbers yield diminished heat transfer with an exception for the case of $\mathrm{Pr} = 10$ and $\mathrm{Ra} = 200$. Via comparing the quantitative results for circular and flat channels, it was demonstrated that the shape of channel cross-section makes significant influence on the average Nusselt numbers, so that the results for one channel cannot be extrapolated over the other one. It was also proved that mixed convection in circular vertical microchannels can be successfully modelled using the LBM methodology, whose deviation from the analytical solution is less than 1%.

1. Introduction

Simulations of microchannel flows attract much interest of scientists and engineers over the last decades inspired by the development of various microelectromechanical and microenergy systems, micro- and nanofabrication, as well as nanotechnology [1–4]. Measurements and modeling in microdevices involve microscales, whose characteristic length lies in the range of micrometers. The scaling laws used in common-size mechanical engineering, turbomachinery etc. are often not applicable to physical phenomena in gas flows in microsystems affected not by rarefaction of the gases due to reduced density, but by reduced length scales of the channels [5].

The rarefaction effects in microchannels can be quantified using the Knudsen number Kn, a dimensionless criterium proportional to the ratio of the mean free path of a gas molecule, L, and a characteristic length of the channel cross section L_{ref}. From the physical point of view, the Knudsen number outlines slippage effects at the channel wall, including the velocity and temperature jump at the wall (see the slip boundary conditions in the section “Governing equations” below). The Knudsen number used in the present work is defined below in the section “Governing equations” and includes the viscous slip coefficient, which takes into consideration the accommodation processes on the surface. The modern theory of the viscous slip coefficient is outlined in details in the works of Sharipov and Seleznev [6], Sharipov [7], and Agrawal and Prabhu [8]. In other words, the Knudsen number makes it possible to model sophisticated fluid flow and thermal processes on the fluid-solid interface in microflows.

For $\mathrm{Kn} \le 10^{-2}$, fluid flow can be modeled using the Navier—-Stokes equations in combination with no-slip boundary conditions [9]. The onset of the slip-flow regime occurs for $10^{-2} \le \mathrm{Kn} \le 10^{-1}$. However, these limits are not strictly defined; for instance, the range of $10^{-3} \le \mathrm{Kn} \le 10^{-1}$ for slip flow conditions was suggested in Ref. [10]. For larger Knudsen numbers $10^{-1} \le \mathrm{Kn} \le 10$, transition regime arises, where the continuum assumption does not hold any

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Nomenclature

A	axial temperature gradient on the channel wall
c	molecular velocity
c_p	specific heat at constant pressure
d	space dimension
F	external force term
f	molecule distribution function
f_T	energy distribution functions
g	gravitational acceleration
G	Archimedes force
$\mathbf{k}$	unit vector directed streamwise
L	free path of a gas molecule
p	pressure
q	heat flux
r, z	cylindrical coordinates
r_0	channel radius
R_g	gas constant
S	channel cross-section
t	time
T	temperature
u	axial velocity component
$\mathbf{u}$	velocity vector

Greek symbols

α	thermal diffusivity
β	coefficient of thermal expansion
γ	viscous slip coefficient
ε	internal energy
μ	dynamic viscosity
η	dimensionless radial variable
Π	channel perimeter
ρ	density
ϕ	relative temperature

Dimensionless values

E	parameter of the pressure gradient
N_n	number of elements in the lattice column across the channel
Nu	Nusselt number
Kn	Knudsen number
Ra	Rayleigh number
Pr	Prandtl number
U	dimensionless axial velocity
R	dimensionless radial coordinate
Θ	dimensionless temperature
τ	dimensionless relaxation time

Subscripts

0	case Ra = 0
00	case Ra = Kn = 0
j	velocity directions
T	energy
w	wall
*	incompressible medium

Superscripts

e	equilibrium distributions

Acronyms

BGK	Bhatnagar-Gross-Krook
CFD	Computational Fluid Dynamics
LBM	Lattice Boltzmann method
2D	two-dimensional
3D	three-dimensional

more. This regime can be modeled using the direct Monte Carlo simulation approach; in doing so, the Burnett equations [11] become necessary.

The heat flux at a solid–gas interface in microchannels induces a temperature jump defined as [12]

$$\Delta T_i \sim L\left(\frac{\partial T}{\partial y}\right)_{y=0}, \tag{1}$$

where L is proportional to the interfacial thermal resistance length l_k known also as Kapitza length discovered first in 1941 [13].

Neumann and Rohrmann [14] modeled flows in a finite range of the Knudsen number for slip and transition flows using the Lattice Boltzmann method within the Peano framework. Further, the Lattice Boltzmann solver was applied to a microreactor build up of differently sized channels and a reactor chamber. A simple Bhatnagar-Gross-Krook (BGK) collision kernel was applied in coarse grid regions situated rather far from the slip boundaries. The computed results agree well with the theory and non-adaptive simulations.

Avramenko et al. [15–17] simulated straight and curved microchannel geometries with isothermal steady-state and start-up flows and applied both analytical and LBM methodology. The latter demonstrated an excellent agreement with the analytical approach. The studies involved detailed insight into effects of the Knudsen number and channel curvature on the velocity profiles and hydraulic resistance of the channel.

Combined free and forced convection in channels have been studied for many decades. Different channel geometries and physical effects have been investigated both analytically and numerically. Galanis and Behzadmehr [18] published a review of studies on mixed convection in vertical ducts covering 20 years backwards from the year 2008. The review involves experimental, analytical and numerical studies that reveal complexity of such flows as compared with pure forced convection. In mixed convection flows, one needs to appropriately modify velocity and temperature profiles, to account for possible reverse flow at different cross-sections and significant buoyancy effects on the flow regime, heat transfer and friction.

In a more recent review, Dawood et al. [19] discussed the last advances in investigations of free, forced and mixed convection in annular channels with nanofluids, as well as effects of eccentricity in horizontal, inclined and vertical channels. Authors also looked into effects of the heater length, Darcy, Prandtl, Reynolds, Grashof and Rayleigh numbers.

Avci and Aydin [20] obtained an analytical solution for fully developed mixed convective heat transfer at fully developed flow of a Newtonian fluid in a vertical parallel plate microchannel with account for the velocity slip and the temperature jump at the wall. One wall was hotter, and the other one was colder than fluid itself. The authors studied effects of the Grashof and Knudsen numbers, as well as the ratio of wall temperature difference between the fluid and both differently heated walls. An empirical relation for the Nusselt number was proposed. The drawbacks of the work [20] are (i) constant values of the wall temperatures, and (ii) simplified approximations of the temperature profile (as a linear function)