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Transient thermal stress intensity factors for a circumferential crack in a hollow cylinder based on generalized fractional heat conduction

X.-Y. Zhang, X.-F. Li*

School of Civil Engineering, Central South University, Changsha 410075, PR China

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A B S T R A C T

A generalized fractional heat conduction theory is applied to investigate the transient thermal fracture problem of a hollow cylinder with an embedded or surface circumferential crack. Integral transform technique is used to solve an associated initial-boundary value problem. Explicit temperature field and thermal stresses are given in the Laplace transform domain for a circumferentially cracked hollow cylinder subjected to thermal shock at the inner surface and with an insulated outer surface. Numerical results in the time domain are obtained by using numerical inversion of the Laplace transform. Transient thermal stresses induced by a crack are determined and thermal stress intensity factors at the crack front are calculated. The effects of fractional order, phase lag of heat flux on the transient temperature field, thermal stresses and thermal stress intensity factors are illustrated graphically for internal and surface cracks. The obtained results based on non-Fourier law of fractional heat conduction are compared with those using the classical Fourier law and hyperbolic heat conduction models, respectively.

1. Introduction

Thermal stress analysis of vessels and pipes is of particular interest, especially for those with defects like cracks. The presence of cracks results in very high thermal stresses near the cracks due to change in environment temperature, and this may cause catastrophic failure of a cracked structure. The most famous example is the explosion of Challenger shuttle after several seconds of ignition in 1986. Such vessels and pipes with crack are frequently encountered in engineering applications. A typical case is a hollow cylinder with embedded or surface crack subjected to heat shock and a great deal of work on fracture of cracked cylinders has been done. For example, Nied and Erdogan [1] analyzed an internal circumferential surface crack of a hollow cylinder suddenly cooled from the inside surface. Furthermore, the effect of cladding on the thermal shock resistance was also dealt with for a circumferentially cracked hollow cylinder [2]. Later, Meshii and Watanabe [3,4] obtained a series of results of stress intensity factors for circumferential cracks under radial temperature distribution or thermal striping. Kordisch et al. [5] studied initiation and growth of a circumferential crack in the HDR-RPV-cylinder under pressurized thermal shock. Dag et al. [6] further formulated circumferential crack problems of a functionally graded material cylinder subjected to temperature change. The finite element method was used to determine an accurate weight function for the crack and a closed-form thermal stress intensity factor with the aid of the weight function method was extracted by Nabavi and Ghajar [7]. For functionally graded cylinders with internal circumferential cracks, the thermal stress intensity factor expression was derived through the weight function method in Ref. [8]. The effect of heat conduction in a penny-shaped crack on thermal stress intensity factors was investigated in Ref. [9].

All of the above-mentioned studies are based on the classical Fourier's law. As well known, although it is widely used in many applications, an evident shortage is that heat diffusion behavior has infinite speed of propagation [10]. That is, if temperature at some point in a medium suddenly changes, it will be felt simultaneously at any other position in the medium. This unphysical hypothesis makes the classical Fourier heat conduction theory unacceptable, in particular for ultrafast heat transfer or extremely high temperature gradient. To overcome this disadvantage, many generalized heat conduction theories have been proposed such as hyperbolic heat conduction model [11,12], dual phase lag heat conduction model [13] and fractional heat conduction model [14], etc.

Within the framework of hyperbolic heat conduction theory, some researchers treated crack problems of cracked elastic media under heat shock (see e.g. Refs. [15,16]). In particular, Fu et al. [17,18] investigated the fracture behavior of a hollow cylinder containing a

* Corresponding author.
E-mail address: xfli@csu.edu.cn (X.-F. Li).

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circumferential crack under convective heat transfer boundary conditions and analyzed the effect of material properties on stress intensity factor of a circumferential crack. As pointed out in Ref. [19], the hyperbolic heat conduction equation predicts physically impossible negative thermal energies and is not a continuation of the classical model at very short time scales.

On the other hand, as a natural extension of the classical integer-order calculus, differential equations of fractional order have been developed to study various problems in science and technique [20,21]. Sherief et al. [22] and Youssef [23] respectively proposed new theories of thermoelasticity and generalized thermoelasticity using the methodology of fractional calculus with one relaxation time. Povstenko [24] considered theories of thermoelasticity based on all four fractional Cattaneo telegraph equations which are proposed in Ref. [25]. Due to its non-local property, implying that the next state of a system depends upon not only its current state but also all of its historical states, Qi and Liu [26] derived an analytical solution corresponding to the time-fractional radial heat conduction in some hollow geometries. Ezzat and EI-Bary [27] applied a fractional model to solve a problem of an infinite long hollow cylinder in the presence of an axial uniform magnetic field. Many representative results on fractional heat conduction in connection with thermal stresses can be found in Ref. [28]. These studies are limited to elastic media without defect. When some defects such as crack are present in media, how the fractional heat conduction affects thermal stresses is interesting. For an elastic layer with embedded and edge cracks, the effect of fractional order on thermal stresses was investigated recently [29]. Nonetheless, for a hollow cylinder with a circumferential embedded or surface crack, when the fractional heat conduction theory is adopted under heat shock at the cylinder surface, there is no information available on transient singular thermal stresses induced by crack.

The aim of this paper is to investigate transient thermal stresses in a hollow cylinder with a circumferential crack based on the generalized fractional thermoelasticity theory. The thermal stress intensity factors (SIFs) of embedded and surface cracks are determined. For a hollow cylinder subjected to heat shock at the surface, transient temperature field and thermal stresses are calculated using numerical inversion of the Laplace transform. The thermal stresses induced by crack along with the corresponding SIFs are obtained through the singular integral equation method. The influences of fractional order α and the phase lag of heat flux τ_q on the transient temperature, thermal stresses and thermal SIFs are analyzed for embedded and inner/outer surface cracks, respectively. Obtained numerical results are compared with those using the classical Fourier and hyperbolic heat conduction theories.

2. Formulation of the problem

Consider a hollow homogeneous isotropic thermoelastic cylinder with a circumferential crack, as shown in Fig. 1. The inner and outer radii of the cylinder are denoted as $r = a$ and $r = b$, respectively. The circumferential crack lies in the plane $z = 0$ and occupies the region $c < r < d$. Here we have used the cylindrical polar coordinate (r, θ, z). For simplicity, we consider an axisymmetric problem. Assume that initial temperature of the cylinder is T_0, the inner surface of the cylinder suffers a thermal shock T_a, and the outer surface is insulated. Then the boundary conditions can be expressed as

$$T(a,t) = T_a H(t), \tag{1}$$

$$\frac{\partial T(b,t)}{\partial r} = 0, \tag{2}$$

where $H(t)$ denotes the Heaviside unit step function. Under such thermal shock, we will analyze transient thermal stress response due to the presence of a circumferential crack. For heat conduction problems, in the present paper a non-Fourier generalized fractional heat conduction law is adopted. Once transient temperature field is determined, thermal stresses induced by crack can be obtained by superposition under the assumption that crack and deformation do not change temperature distribution. Since singular thermal stresses are of particular interest, the corresponding thermal SIFs will be evaluated. Therefore, the problem is decomposed to two subproblems.

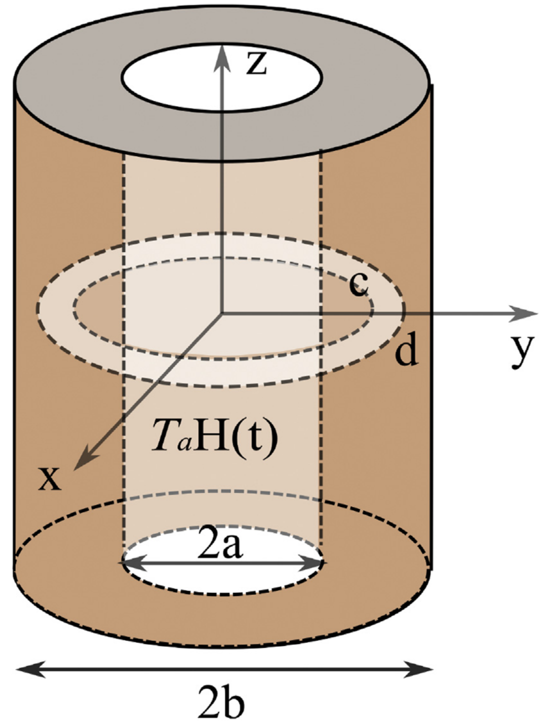

Fig. 1. A hollow cylinder containing a circumferential crack.

3. Thermal stresses in a hollow cylinder without crack

First, let us determine transient thermal stresses in a hollow cylinder without crack. To this end, it is sufficient to look for transient temperature in this cylinder subject to thermal shock. Applying fractional Taylor's series developed by Jumarie [30], we expand heat flux vector $\mathbf{q}(\mathbf{x}, t+\tau_q)$ up to fractional order α in phase lag time of heat flux τ_q to result in Refs. [31,32].

$$\mathbf{q}(\mathbf{x},t) + \frac{\tau_q^\alpha}{\alpha!}\frac{\partial^\alpha \mathbf{q}(\mathbf{x},t)}{\partial t^\alpha} = -k\nabla T(\mathbf{x}), \tag{3}$$

where k and T are thermal conductivity and absolute temperature, respectively, ∇ is the gradient operator, and the time-fractional differential operator $\partial^\alpha/\partial t^\alpha$ with $0 < \alpha \le 1$ is defined by

$$\frac{\partial^\alpha f(*,t)}{\partial t^\alpha} = \begin{cases} f(*,t) - f(*,0), & \alpha \to 0 \\ I^{1-\alpha}\dfrac{\partial f(*,t)}{\partial t}, & 0<\alpha<1 \\ \dfrac{\partial f(*,t)}{\partial t}, & \alpha = 1, \end{cases} \tag{4}$$

with