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Optimum interfaces that maximize the heat transfer rate between two conforming conductive media

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We consider conjugate heat transfer between two conductive and conforming media, with isothermal boundary conditions on the exposed surfaces, and continuity of the temperature and the heat flux along their interface. We address the inverse problem of finding the shape of the interface such that the heat transfer rate is maximized. We formulate three isoperimetric, shape optimization problems associated with three different applications: i) the optimal shape of corrugations (surface "roughness"), ii) the optimal shape of high conductivity inserts (inverted fins) and iii) the optimal shape of high conductivity fins. As expected, the optimal geometries have the shape of an extension of the high conductivity material into the low conductivity material. For the case of corrugations and inserts, the optimum shapes are triangular for small perimeters; for large perimeters and thick slabs they are elliptical and tend to cover the whole width/period of the domain. Optimum fins are characterized by long, shallow valleys and deep, narrow protrusions of the high conductivity material. For the parameters considered in this study, the width of the protrusion is approximately one quarter of the period.

1. Introduction

In our work we consider heat transfer in a composite wall which consists of two different materials in series (Fig. 1). For a flat interface the heat conduction is one-dimensional and the heat transfer problem can be addressed using the concept of thermal resistance to obtain the heat transfer rate. Basically, the analysis assumes one-dimensional heat conduction in each layer ($d^2T/dx^2 = 0$), and conjugate heat transfer across layers, i.e. it requires continuity of temperature and flux across the interface of the two materials/media:

$$T_1 = T_2 \quad \text{and} \quad k_1 \frac{\partial T_1}{\partial x} = k_2 \frac{\partial T_2}{\partial x}.$$

If one of the media is a fluid, then the analysis would require to apply the convection-diffusion equation in this particular medium instead of $d^2T/dx^2 = 0$. However, the convection problem is simplified by modeling convection as a boundary condition to the conduction problem. This is achieved by assuming that the heat flux is proportional to the temperature difference between the surface and the far field [1], i.e.

$$k \frac{\partial T}{\partial n} = h \left(T_\infty - T_{\text{surface}} \right), \tag{1}$$

where the convection heat transfer coefficient (h) is assumed constant and obtained independently by analytical, numerical and experimental methods. The above analysis can be easily extended to radial and spherical systems, and it can also provide approximate results for two-dimensional configurations when, for example, the composite wall involves parallel layers of different materials. Furthermore, it is standard textbook approach to use one dimensional approximation to develop the governing heat transfer equations associated with extended surfaces [1]. The one-dimensional approach has been also used to obtain the optimal shape of extended surfaces/fins [2–4]. However, it is established in the literature that a one-dimensional heat transfer approximation, when the underlying heat transfer process is two-dimensional, conceals some very interesting phenomena: the existence of a critical thickness for planar configurations [5,6], the existence of a critical Biot number associated with extended surfaces [7–9], and the existence of a critical depth [6,10,11] associated with buried

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Nomenclature

H	thickness of the composite wall (dimensionless)
H_1	thickness of medium 1 (lower medium) (dimensionless)
h	convection heat transfer coefficient $(\mathrm{W}/(\mathrm{m}^2\ \mathrm{K}))$
k	thermal conductivity (W/(m K))
P	perimeter of the interface (dimensionless)
L	periodic length of the geometry (m); Length scale used for non-dimensionalization
S	shape factor (dimensionless)
T	temperature (dimensionless)
x, y	coordinates of the physical plane (dimensionless)

Greek symbols

$\lambda = k_1/k_2$ ratio of conductivities of the two media

pipes.

While direct problems deal with the solution of a governing partial differential equation associated with a certain process, i.e. the Laplace equation with appropriate boundary conditions in the case of heat conduction [12], in inverse problems the relevant partial differential equation appears as a constraint to an optimization problem. For example, inverse problems dealing with heat conduction can be associated with the estimation of an unknown boundary heat flux by using temperature measurements taken below the boundary surface. Another example is the determination of the boundary conditions, the physical properties, the geometrical configuration of the heated body and the heat flux, by knowing the temperature distribution on some part of the boundary of the heat conducting body [13,14]. The objective function is the difference between the computed temperature and the measured/prescribed temperature [15,16]. In Refs. [15,16], the authors used a body-fitted grid generation to map the unknown optimal shape onto a fixed computational domain, where the heat conduction problem was solved using finite differences, and an efficient sensitivity analysis that is expressed explicitly in the fixed computational domain. Finally, the Conjugate Gradient method is used to minimize the objective function. A similar approach was used by Cheng et al. [17,18], however they have included convection heat transfer.

While in the Shape Optimization problems mentioned above, the unknown shape has a predefined configuration, for example a simply connected domain, the more general case where the design can attain any form in the design space, as it allows changes not only in shape but also in the topology of the target structure, is the subject of Topology Optimization [19–26]. The most popular method used in Topology Optimization problems, is the level-set method [27]. Topology Optimization problems, and in general inverse problems, are typically ill-posed because of their sensitivity to random errors in the measured data and/or in the numerical solution of the underlying partial differential equation, especially if the inverse problem involves the estimation of a large number of parameters. To overcome this problem a regularization (Tikhonov regularization) [28,29] or a homogenization [30] method is often used.

Inverse shape optimization problems are usually solved using a parameter estimation approach by assuming a functional form for the unknown shape, e.g., isogeometric analysis [31], mesh-morphing [32], eigenfunction expansion [13], boundary control points [33] and adaptive mesh [34]. Similar to [15–18], where the physical domain is mapped onto a fixed computational domain, another approach that was successful in obtaining optimal shapes that maximize the heat transfer is to use conformal mapping techniques [6,7,35–39]. The Shape Optimization problem is formulated as a nonlinear programming problem (constrained nonlinear optimization), i.e. find the constrained extremum of a scalar function of several variables, where the variables are the parameters of the generalized Schwarz-Christoffel transformation. The Shape Optimization problem mentioned above is solved numerically using the NLPQL code developed by Schittkowski [40], which uses a special implementation of a sequential quadratic programming (SQP) method. To generate a search direction a quadratic subproblem is formulated and solved. The line search can be performed with respect to two alternative merit functions, and the Hessian approximation is updated by a modified BFGS formula [41].

As mentioned in the beginning, in this work we will address the problem of finding the optimal interface between two conductive media. We will assume that the exposed sides are isothermal and that along the polygonal interface the two media are in perfect

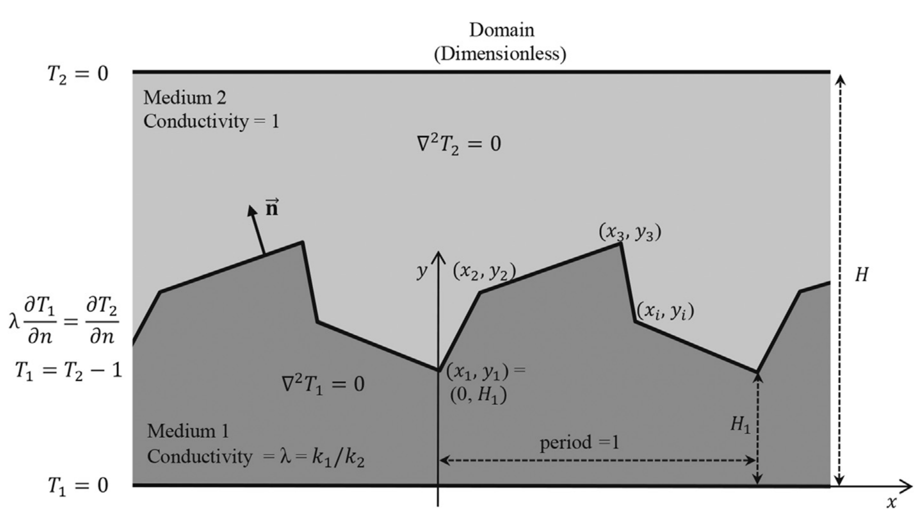

Fig. 1. Schematic representation of the problem and boundary conditions. All variables are non-dimensionalized with the period, hence the non-dimensionalized period is 1 (one). The normal vector is denoted as $\vec{\mathbf{n}}$. The shape of the periodic interface is assumed unknown and we seek the shape that maximizes heat transfer. Note that the interface is discretized in line segments.