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# A reactive hydromagnetic heat generating fluid flow with thermal radiation within porous channel with symmetrical convective cooling

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ABSTRACT

The thermodynamics analysis of a reactive hydromagnetic radiative heat transfer flow within a channel filled with saturated non−Darcy porous medium with convective cooling of the walls is investigated for Arrhenius kinetics. The momentum and energy equations governing the fluid flow are modeled, non-dimensionalised and solved analytically by making use of modified Adomian decomposition Method (MADM). The expressions of momentum and energy profiles are used to analysed the entropy generation rate and the impacts of other flow thermophysical parameters especially the radiative flux on the fluid flow including the thermal stability analysis obtained using Padé approximation technique are presented and discussed.



## 1. Introduction

Due to the diversity of fluid in nature, studies through investigation explained that lots of models have been proposed to describe fluid behaviour in different circumstances as in Ref. [1]. Studies involving flow of reactive hydromagnetic fluid have been investigated in Refs. [2–8] because of its extensive scope in engineering and industrial applications such as electronic cooling, thermal insulation, crude oil extraction and nuclear reactor. Also, studies involving reacting materials undergoing exothermic reaction and Newtonian cooling where convection forms an integral part of heat transfer due to differences in ambient temperatures as described in Refs. [2–4,9–11]. Meanwhile, the process of convective heat transfer has been examined in several studies mentioned in Refs. [10–16] which involve various flow of fluid between walls with convective cooling effects have been investigated because of its importance in new technological applications, for instance, the cooling processes of nuclear reactor and refrigerators.

Additionally, recent discoveries of magnetic impact on fluid flow cannot be neglected due to the fact that, the magnetic strength placed in a transverse direction within the channel undergo series of interactions especially in controlling hot moving fluid. For example [17], investigated the impact of magnetic source on nanofluid hydrothermal treatment in an enclosure with square hot cylinder. Also [18,19], considered the impact of induced magnetic field in the process of heat and mass transfer for nanofluid using Buongiorno model. In addition to that [20], examined the influence of magnetic field dependent (MFD) viscosity on MHD nanofield flow and heat transfer, thereby concluding that a reduction in heat transfer due to MFD viscosity is a rising function of Rayleigh number but a reducing function of magnetic strength parameter.

However, a lot of attention has been devoted to the study involving the impact of thermal radiation on fliud flow, like in Ref. [21], the study stated that it plays a vital role in the context of space technology and especially in a process involving high temperature. Other investigations that revealed the marked effect of thermal radiation, to mention few, are described in Refs. [16,18,19,21–25]. Also, the thermal stability analysis is another aspect of fluid flow that cannot be ignored because it gives information on the prediction of the critical or unsafe flow conditions as extensively explained in Ref. [26]. In support of this [27], studied

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the steady state solutions for viscous reactive flows through channels with a sliding wall obtaining the analysis using a special type of Hermite–Padé approximation approach which has been proved extremely useful in the validation of purely numerical scheme. Other relevant studies showing the significance of thermal stability analysis can be found in Refs. [28–31].

Hence, the present study is to examine the thermodynamics analysis of a radiative heat transfer of a reactive hydromagnetic fluid through parallel porous plates under the effects of heat source and thermal radiation with convective boundary conditions following Newton's cooling law. This study also spreads out the recent work of [2–7] to examine the marked effect of thermal radiative heat transfer on the flow system with convective cooling of the walls using the modified Adomian decomposition method (MADM) to obtain the solutions of the momentum and energy equations governing the fluid flow. The choice of this method is due to the fact that, the method does not demand any linearization, discretization and use of guess or perturbation. This method, from literature has been proved to be efficient, reliable and a powerful tool in providing solution of differential and integral equations in a rapidly convergent series as discussed extensively in Refs. [32–36] and that it assures size-able savings in the computational volume. In addition to that, the analysis of thermal stability of the flow system is obtained using Padé approximation technique as obtained in Refs. [4–6,31].

In the rest of this paper, the mathematical model of the flow system are formulated in section 2. The non–linear equations for momentum and energy are solved in section 3 by making use of MADM and determine the entropy generation rate from the expressions of velocity and temperature profiles. The analysis of thermal stability is presented in section 4, the graphical presentation of results are shown in section 5 while the final conclusion was done in section 6.

## 2. Mathematical model

We consider a steady flow of an incompressible internal heat generating flow of a reactive hydromagnetic fluid within parallel porous plates of distance (2a) located at $y=-a$ and $y=a$ under the influence of radiative flux with convective cooling of the walls as depicted in the figure below (Fig. 1). In this present work, we neglect the consumption of the reactant of which the momentum and energy equations governing the fluid flow are given in non–dimensionless forms as mentioned in Refs. [2–5,10] may be written as:

$$\mu\frac{d^2\overline{u}}{d^2\overline{y}}\overline{u}-\frac{d\overline{P}}{d\overline{x}}-\frac{\mu}{K}\overline{u}-\sigma_0 B_0^2=0 \tag{1}$$

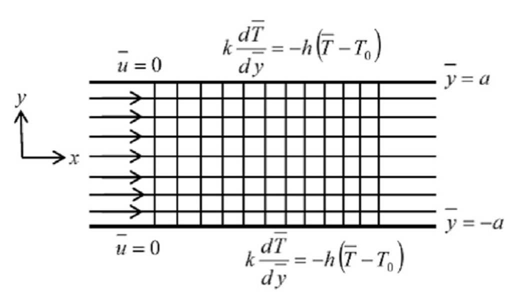

**Fig. 1.** The physical geometry of the flow regime.

$$k\frac{d^2\overline{T}}{d^2\overline{y}}+QC_0Ae^{-\frac{E}{R\overline{T}}}+\mu\left(\frac{d\overline{u}}{d\overline{y}}\right)^2+\frac{\mu}{K}\overline{u}^2+\sigma_0 B_0^2\overline{u}^2+Q_0\left(\overline{T}-T_0\right)-\frac{dq_r}{d\overline{y}}=0 \tag{2}$$

with symmetric condition along the channel centreline given as

$$\frac{d\overline{u}}{d\overline{u}}=\frac{d\overline{T}}{d\overline{u}}=0 \quad \text{on} \quad y=0 \quad \text{and} \quad \overline{u}=0, \quad k\frac{d\overline{T}}{d\overline{y}}=-h\left(\overline{T}-T_0\right) \quad \text{on} \quad y=a \tag{3}$$

such that $P$ represents the pressure, $T$ represents the fluid temperature, $\mu$ is known to be the fluid viscosity, $u$ is the velocity, $\sigma_0$ represents electrical conductivity and $K$ is Darcy's permeability constant, $k$ represents the thermal conductivity coefficient, $T_0$ is the wall temperature, $A$ is the constant of reaction rate, $h$ is the heat transfer coefficient, $Q$ is the heat of the reaction term, $C_0$ denotes the initial concentration of reactant species and $E$ is the activation energy. In addition to that, $R$ denotes the universal gas constant, $Q_0$ represent the dimensional heat generation coefficient and $q_r$ denoted the radiative heat transfer flux.

The additional last term in the velocity equation (1) and the fifth term in energy equation (2) marked the impact of the magnetic field strength as in Refs. [2–8,17–20]. Moreso, the sixth term in energy equation (2) is the internal heat generation within the flow system as in Refs. [6,7,37,38] while the last term is the significant effect of radiative heat transfer fluid flow as described in Refs. [16,18,19,22–24]. The Rosseland approximation for thermal radiation is given as:

$$q_r=-\frac{4\sigma}{3k^*}\frac{d\overline{T}^4}{d\overline{y}} \tag{4}$$

such that $\sigma$ denotes the Stefan-Boltzmann constant and $k^*$ represent the mean absorption coefficient. A general assumption with the temperature difference for the flow system is such that $T^4$ may be expanded in Taylor series about the free-stream temperature, $T_\infty$ and by neglecting the higher orders as done in Refs. [23,24] yield:

$$T^4 \equiv 4T_\infty^3 T-3T_\infty^4 \tag{5}$$

such that

$$\frac{dq_r}{d\overline{y}}=-\frac{16\sigma T_\infty^3}{3k^*}\frac{d^2\overline{T}}{d\overline{y}^2} \tag{6}$$

However, the entropy generation rate ($S^m$), due to heat transfer under the influence of considerable radiative heat flux and the compound effects of fluid resistance on Joules dissipation, porous medium and magnetic field strength following [3,23,24,39–42] is given as:

$$S^m=\frac{k}{T_0^2}\left[\left(\frac{d\overline{T}}{d\overline{y}}\right)^2+\frac{16\sigma T_\infty^3}{3kk^*}\left(\frac{d\overline{T}}{d\overline{y}}\right)^2\right]+\frac{1}{T_0}\left(\mu\frac{d\overline{u}^2}{d\overline{y}}+\frac{\mu}{K}\overline{u}^2+\sigma_0 B_0\overline{u}^2\right) \tag{7}$$

We introduce these non-dimensional quantities in (1)–(7)