



Energy principles for temperature varied with time



Zhen-Bang Kuang

Shanghai Jiaotong University, Shanghai, 200240, PR China

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ABSTRACT

This paper discusses some extensions of energy principles in classical and continuum thermodynamics. The modified energy principles will extend the first and second laws in classical continuum thermodynamics to the case where the temperature is varied with time. It is shown that the modified energy principles are consistent with the statistical mechanics. The inertial heat and inertial entropy concepts play the kernel roles in the modified energy principles. The temperature wave equation with finite phase velocity is a natural result of the inertial entropy theory. The effect of the temperature inertia in multi-field coupling case is discussed shortly.

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1. Introduction

When the temperature is varied in space, the irreversible heat conduction is discussed extensively [1–3]. When the temperature varies with time, in the classical heat conduction theory the temperature propagates subjected to the parabolic differential equation and with infinite phase speed. However the second sound speed observed by experiments in liquid helium [4] and later in solids [5] at low temperature urges the development of the hyperbolic temperature wave equation with finite phase speed. The applications of short pulse lasers in the fabrication of sophisticated microstructures, syntheses of advanced materials and measurements of the thin-film properties [6] and applications in bioengineering [7] are the new active research areas. In the problems with microscopic size and time, especially at low temperature, the temperature satisfies the wave equation. Therefore many generalized thermodynamic theories, especially the generalized heat conduction theories, were proposed to allow a finite propagation speed for the temperature [7,8].

In this paper the following notations are used: a comma followed by index i indicates partial differentiation with respect to x_i , such as $u_{i,j} = \partial u_i / \partial x_j$; a superimposed dot indicates partial differentiation with respect to time, such as $\dot{u} = du/dt$. For the small deformation case $\partial(\cdot)/\partial t = d(\cdot)/dt$. In this paper the Einstein summation convention, i.e. repeated subscripts of a variable imply

summation over the range of that subscript, such as $u_{i,i} = u_{1,1} + u_{2,2} + u_{3,3}$ in three dimensional space, is also used.

For the isotropic media, the first temperature wave theory was proposed by Cattaneo [9,10] and then by Vernotte [11]. They proposed the following C-V heat conduction equation:

$$q_i + \tau_0 \dot{q}_i = -\lambda \vartheta_{,i} \quad (1)$$

where \mathbf{q} is the heat flow vector, τ_0 is a relax time constant, $\vartheta = T - T_0$, where T is the Kelvin temperature and T_0 is a reference temperature, such as the environment temperature, λ is the heat conduction coefficient. In usual pure heat conduction theory we have

$$\dot{u} = \dot{\gamma} - q_{i,i} = C \dot{\vartheta} \quad (2)$$

where u is the internal energy, C is the specific heat per volume and γ is the internal heat source strength. Combining Eqs. (1) and (2) we get a hyperbolic equation for temperature:

$$C(\dot{\vartheta} + \tau_0 \ddot{\vartheta}) = \dot{\gamma} + \tau_0 \ddot{\gamma} + \lambda \vartheta_{,ii} \quad (3)$$

C-V theory is applied extensively. From Eqs. (1) and (2) it can be seen that the entropy density is not a state function which is not consistent with the classical thermodynamics. Jou et al. [12] proposed the extended irreversible thermodynamics to mend this difficulty and other problem in nonequilibrium thermodynamics by modifying the classical entropy formula and introducing the

E-mail address: ZbKuang@mail.sjtu.edu.cn.

nonequilibrium temperature.

Green and Laws [13] used a new temperature function $\phi = \phi(T, \dot{T})$, $T = \phi(T, 0)$ instead of the Kelvin temperature T , but the concrete relation is related to the other assumptions. They use a new free energy $g = u - \phi s$ and modified the Clausius-Duhem inequality as $\int_V \dot{s} dV - \int_V (\gamma/\phi) dV + \int_a (q_i/\phi) n_i da \geq 0$, where \mathbf{n} is the outer normal of a surface a of the volume V , s is the entropy density per volume. On this basis Green and his coworker discussed the thermoelasticity under finite deformation. After simplification they proposed a temperature wave equation with two parameters. Obviously the free energy g in the Green- Laws theory is not a state function and is also not consistent with the classical thermodynamics.

Guo et al. [14] proposed the microscopic thermomass theory based on the assumption that the thermal energy is equivalent to a small “thermomass” and the Einstein’s mass-energy equivalence relation is used. In dielectric materials they treated the thermomass as the phonon gas and the heat transport is considered as the motion of phonon gas. Using this theory they derived the generalized heat conduction law and the nonequilibrium temperature.

There are also several other theories for the temperature wave equation. These theories in most of literature were mainly established through modified Fourier heat conduction equation or introduced new temperature function and modified the Clausius-Duhem inequality [7,8].

In Eq. (1) the term containing τ_0 is a viscous term for the heat flow-temperature equation, however in the temperature equation Eq. (3) the term containing τ_0 is obviously an inertial term. Such being the case, why we do not directly introduce the concept of the temperature inertia to simplified the problem? In literature [15–17] and some other papers Kuang proposed a new inertial entropy theory (IET), directly introducing the temperature inertia to derive the temperature wave equation. This method is based on the universal modified classical thermodynamic principles.

2. Modified energy principles and inertial entropy

The classical thermodynamics discusses thermal process in an equilibrium state, i.e. the temperature is constant in the four dimensional (space-time) space. The first classical thermodynamic law is

$$\begin{aligned} \dot{U} &= \dot{W} + \dot{Q}, \quad \dot{W} = \int_V \mathbf{f} \cdot \mathbf{v} dV + \int_{a_\sigma} \mathbf{T} \cdot \mathbf{v} da \\ \dot{U} &= \int_V \dot{u} dV, \quad \dot{Q} = - \int_{a_q} \mathbf{q} \cdot \mathbf{n} da + \int_V \dot{\gamma} dV, \quad \mathbf{v} = \dot{\mathbf{u}} \end{aligned} \quad (4)$$

where U, W and Q are internal energy, work done by the external force and heat supplied by the environment respectively, \mathbf{v} is the velocity, \mathbf{u} is the displacement vector. \mathbf{f} is the body force, \mathbf{T} is the surface traction, a_σ and a_q are respectively the parts giving external force and heat flow of a .

The second classical thermodynamic law is

$$S_b - S_a \geq \int_a^b \frac{dQ}{T}, \quad S = \int_V s dV \quad (5)$$

where S is the entropy in the equilibrium state, S_b and S_a are the values of the entropy at states a and b , s is the entropy density per volume. In Eq. (5) the symbol “=” is used for the reversible process and the symbol “>” is for the irreversible process.

In classical thermodynamics for a moving body Eq. (4) is changed to

$$\dot{U} + \dot{K} = \dot{W} + \dot{Q}, \quad K = (1/2) \int_V \rho \mathbf{v} \cdot \mathbf{v} dV \quad (6)$$

where K is the kinetic energy. Eq. (6) can also be written as

$$\dot{U} = \dot{W} - \dot{W}^{(a)} + \dot{Q}, \quad \dot{W}^{(a)} = \int_V \rho \dot{\mathbf{u}} \cdot \mathbf{v} dV \quad (7)$$

where $\rho \dot{\mathbf{u}}$ is the inertial force according to the D’Alembert’s principle and $\dot{W}^{(a)}$ is the work rate done by the inertial force.

In literature [15–17] the author considered that the variation of temperature with time should be supplied extra heat from the environment to overcome the heat or temperature inertia. From this idea the temperature wave equation will be derived easily and the definition of usual temperature and the Fourier’s law are still kept. Though many literature have been pointed out that “heat may have inertia”, such as Nernst [18], Onsager [19], Kaliski [20], but these authors only used this concept indirectly through the generalized heat conducting equation.

When the temperature varies with time, in literature [15–17] the author proposed to modify the first thermodynamic law expressed in Eq. (7) to

$$\dot{U} = \dot{W} - \dot{W}^{(a)} + \dot{Q} - \dot{Q}^{(a)} \quad (8)$$

where $Q^{(a)}$ is the inertial heat. Eq. (8) shows that the supplied heat δQ by the surrounding cannot be entirely absorbed by the system immediately, but only $\delta Q - \delta Q^{(a)}$, where $\delta Q^{(a)}$ is used to overcome the temperature inertia. $\delta Q^{(a)}$ may be positive or negative. In Refs. [15–17] author considered that the inertial heat is induced by the temperature inertia, so assumed

$$\begin{aligned} \dot{Q}^{(a)} &= \int_V C \rho_{s0} \ddot{T} dV = \int_V \rho_s \ddot{T} dV \\ Q^{(a)} &= \int_V C \rho_{s0} \dot{T} dV \end{aligned} \quad (9)$$

where ρ_{s0} is an inertial entropy time constant and $\rho_s = C \rho_{s0}$. Eq. (9) shows that, when $\dot{T} > 0$, $Q^{(a)} > 0$, i.e. when the temperature increases, the inertial heat is positive; on the contrary, $\dot{T} < 0$, $Q^{(a)} < 0$.

Correspondingly we can introduce the concept of the inertial entropy rate $\dot{S}^{(a)}$ as

$$\begin{aligned} \dot{Q}^{(a)} &= T \dot{S}^{(a)} = \int_V T \dot{s}^{(a)} dV, \\ T \dot{S}^{(a)} &= C \rho_{s0} \ddot{T}, \quad s^{(a)} = \int_0^t \dot{s}^{(a)} dt \end{aligned} \quad (10)$$

where $s^{(a)}$ is the inertial entropy density per volume, which is only related to the temperature variation with time and does not connected with any dissipative process.

The second thermodynamic law can now be modified to

$$S_b - S_a \geq \int_a^b \frac{dQ - dQ^{(a)}}{T} \quad (11)$$

Correspondingly the classical entropy equation is modified to [15–17]

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