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Anisotropic radiation transfer in a two-layer inhomogeneous slab with reflecting boundaries

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ABSTRACT

In this study the integral form of the radiative transfer equation in a two-layer inhomogeneous slab for linearly anisotropic scattering has been solved using the Galerkin-iterative technique. The medium is considered to have internal space dependent energy source, diffusely and specularly reflecting boundaries, interface transmission and diffuse surface source. The reflection, the transmission coefficients, incident energy and the net radiative heat flux through the medium has been calculated for different media. The calculation was carried out for inhomogeneous media of transparent, diffusely and specularly reflecting boundaries with isotropic, forward and backward linear anisotropic scattering. Results obtained for isotropic scattering in homogeneous and inhomogeneous media of source free problem are compared with the previous work and an excellent agreement has been achieved.

1. Introduction

Radiative transfer trough anisotropic scattering medium with reflecting boundaries has numerous important applications in astrophysics, space science and engineering. Nowadays, new fields of study use the radiative transfer phenomena such as biomedical optics, optical tomography and radiation protection [1–8]. Several techniques have been applied to solve the anisotropic radiation transfer in composite slab in both homogeneous and inhomogeneous media [9–14]. The radiative transfer equation is transfer can be expressed in its integral form by integrating over solid angle and its exact solution is solved using different methods [14–17].

Several authors [15,17,18] reported that the Galerkin-iterative technique is used for solving the problem of particle transfer for isotopic scattering in the two-regions homogeneous and an inhomogeneous media of source free problem. In the present work, the same method is used to solve radiative transfer equation for an absorbing, emitting, inhomogeneous, with isotropic and an anisotropic scattering that contains an internal energy source. The coupled integral equations for the two-layer inhomogeneous slab with diffusely and specularly reflecting boundaries are solved. The calculation of incident energy, net radiative heat flux, the reflection and the transmission coefficients were carried out for different optical thickness and exponential single scattering albedo for isotropic and linear anisotropic scattering.

2. Mathematical formulation

Assuming that radiative transfer equation for an absorbing, emitting, inhomogeneous a two-layer slab, with linear anisotropic scattering contains an internal energy source in the form:

$$\left(\mu\frac{\partial}{\partial x}+1\right)I_1(x,\mu)=\frac{\omega_1(x)}{2}\int_{-1}^{1}I_1(x,\mu')\,p(\mu,\mu')\mathrm{d}\mu' +(1-\omega_1(x))n^2I_b, \quad -1\le\mu\le1, \quad \text{and} \quad 0\le x\le a \tag{1}$$

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$$\left(\mu\frac{\partial}{\partial x}+1\right)I_2(x,\mu)=\frac{\omega_2(x)}{2}\int_{-1}^{1}I_2(x,\mu')p(\mu,\mu')d\mu'+(1-\omega_2(x))n^2I_b \qquad -1\le\mu\le 1,\quad \text{and}\quad a\le x\le b \tag{2}$$

where $I(x,\mu)$ is the radiation intensity of the medium, x is the optical distance, μ is the direction cosine of the intensity, $(1-\omega_S(x))n^2I_b, s=1,2$ is the internal energy source [9], n is refractive index, I_b Planck function, ω is the single scattering albedo, and the subscripts 1 and 2 refer to the layer 1 and 2, respectively. The scattering phase function $p(\mu,\mu')$ can be represented by a series expansion of Legendre polynomials $p_m(\mu)$ as [19].

$$p(\mu,\mu')=\sum_{n=0}^{N}a_m p_m(\mu)p_m(\mu'),\quad a_0=1 \tag{3a}$$

which is given for a linear anisotropic scattering approximation by

$$p(\mu,\mu')=1+\bar{a}\mu\mu' \tag{3b}$$

The linear anisotropic coefficient $\bar{a}$ is expressed in terms of the Legendre − polynomial coefficients a_m as [20].

$$\bar{a}=\sum_{m=0}^{\infty}\left[(-1)^m a_{2m+1}(2m)!\right]\Big/\left[2^{2m}m!(m+1)!\right] \tag{3c}$$

The boundary conditions allowed for specular and diffuse reflection, interface transmission and diffuse surface source are taken as [9].

$$I_1(0,\mu)=f_1+\rho_{1s}I_1(0,-\mu)+2\rho_{1d}J_1^-(0),\quad \mu\ge 0, \tag{4a}$$

$$I_1(a,-\mu)=f_2+\gamma_{21}I_2(a,-\mu)+\rho_{2s}I_1(a,\mu)+2\rho_{2d}J_1^+(a),\quad \mu\ge 0, \tag{4b}$$

$$I_2(a,\mu)=f_3+\gamma_{12}I_1(a,\mu)+\rho_{3s}I_2(a,-\mu)+2\rho_{3d}J_2^-(a),\quad \mu\ge 0, \tag{4c}$$

$$I_2(b,-\mu)=f_4+\rho_{4s}I_2(b,\mu)+2\rho_{4d}J_2^+(b),\quad \mu\ge 0, \tag{4d}$$

where f_s represents diffusely emitting sources, ρ_s and ρ_d are the specular and diffuse reflectivities of the boundaries. γ_{ij} is the interface transmissivity when the radiation enters the j layer from i layer and equal to 1- $\rho_s+\rho_d$. A simple schematic of a two layer-slab is represented in Fig. 1.

The partial heat flux $J_s^{\pm}$ is defined as:

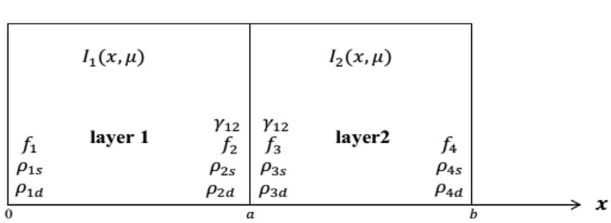

Fig. 1. Schematic of a two layer-slab with illustration of various quantities.

$$J_s^{\pm}(x)=\int_0^1 I_s(x,\pm\mu)\mu\,d\mu,\quad \mu\ge 0 \tag{5}$$

For the first medium, substituting the boundary conditions (4a) and (4b) in Eq. (1) to obtain the integral equations

$$I_1(x,\mu)=I_1(0,\mu)e^{-x/\mu}+\int_0^x\frac{e^{-(x-x')/\mu}}{\mu}\left(G_{01}^*(x')+\bar{a}\mu G_1^*(x')\right)dx',\quad \mu\ge 0 \tag{6}$$

$$I_1(x,-\mu)=I_1(a,-\mu)e^{-(a-x)/\mu}+\int_x^a\frac{e^{-(x'-x)/\mu}}{\mu}\left(G_{01}^*(x')-\bar{a}\mu G_1^*(x')\right)dx',\quad \mu\ge 0 \tag{7}$$

For the second medium, substituting the boundary conditions (4c) and (4d) in Eq. (2) to obtain the integral equations

$$I_2(x,\mu)=I_2(a,\mu)e^{-(x-a)/\mu}+\int_a^x\frac{e^{-(x-x')/\mu}}{\mu}\left(G_{02}^*(x')+\bar{a}\mu G_2^*(x')\right)dx',\quad \mu\ge 0 \tag{8}$$

$$I_2(x,-\mu)=I_2(b,-\mu)e^{-(b-x)/\mu}+\int_x^b\frac{e^{-(x'-x)/\mu}}{\mu}\left(G_{02}^*(x')-\bar{a}\mu G_2^*(x')\right)dx',\quad \mu\ge 0 \tag{9}$$

where

$$G_{0s}^*(x)=\frac{\omega_s(x)}{2}G_{0s}(x)+(1-\omega_s(x)),\quad s=1,2 \tag{10}$$

$$G_s^*(x)=\frac{\omega_s(x)}{2}G_s(x)\quad s=1,2 \tag{11}$$

The irradiance $G_{0s}(x)$ and the net flux $G_s(x)$ are defined as

$$G_{0s}(x)=\int_{-1}^{+1}I_s(x,\mu)d\mu \tag{12}$$