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Convective heat transfer in a porous-medium micro-annulus with effects of the boundary slip and the heat-flux asymmetry: An exact solution

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ABSTRACT

Flow and heat transfer in a micro-annulus with the porous material is analytically investigated using local thermal equilibrium (LTE) and local thermal non-equilibrium (LTNE) models with effects of flow/thermal slips and asymmetric heat fluxes. Analytical solutions for velocity and temperatures are obtained. Effects of key factors on convective heat transfer are examined. An increase in the Knudsen number and the inverse of a modified Darcy number can homogenize the velocity. When heat flux (HF) ratio is $-R_2$, the temperatures of solid and fluid show linear profiles. LTNE model must be considered for small duct scale, lower effective thermal conductivity ratio or poor local convective heat transfer between solid and fluid phases. A limit exists for heat transfer enhancement by homogenizing the flow field. Critical HF ratios corresponding to the asymptotic line are verified. The opposite effects of flow slip and thermal slip on heat transfer lead to the maximum heat transfer. This analytical solution predicts the thermal performance of porous medium micro-annulus in wide ranges of radius ratio, Knudsen number, and HF ratio.

1. Introduction

Flow and thermal transport in porous media have been investigated for a long time with various models proposed. Related applications can be found in many areas, including the crude oil extraction [1], filtration [2], flow of groundwater [3], performance improvement of heat exchanging equipment [4], solar thermal utilization [5], chemical reforming [6], fuel cells [7], heat pipes [8], cooling of nuclear reactors [9], amongst many others.

Volume averaging technique is the most commonly used tool for modeling transport in porous media. The fluid flow in porous media can be studied with Darcy model, Brinkman model or Forchheimer model. The Darcy and Brinkman models are suitable for flows with low velocity (Re_K<<1). In Darcy model, the fluid flow is treated as one-dimensional, while the viscous effect of fluid near the impermeable wall is especially considered in Brinkman model. These two models are generally employed for small velocity. When the fluid velocity in the porous material is very high, the pressure drop is amazing and nonlinearly increased with an increase in velocity. In this case, the Forchheimer model should be employed. Usually, the velocity in porous media is not very high and the combination of Darcy model and Brinkman model is sufficient for modeling such problem. As to thermal transport in porous media, either the local thermal equilibrium (LTE) model or the local thermal non-equilibrium (LTNE) model can be used. The solid phase and the fluid phase of porous media saturated with fluid are regarded as a bulk phase in the LTE model, while the temperature difference between the solid and fluid phases is emphasized in the LTNE model.

Yet, when the fluid thermal conductivity is obviously smaller than the solid thermal conductivity, the LTNE model should be adopted since the LTE assumption overestimates the heat transfer in porous media.

Under the no-slip boundary condition, the heat transfer in porous medium ducts with the normal scale has been extensively investigated. Havstad and Burns [10] numerically calculated the convection in vertical cylindrical annuli with porous media with the perturbation analysis performed. Vafai and Tien [11] theoretically investigated the effects of boundary and inertia term on flow and heat transfer in porous media. Xu et al. [12] used the LTNE model to numerically simulate the heat conduction in a porous medium under the unsteady condition. Wang [13] provided the

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Nomenclature

a_{sf}	specific surface area, m^{-1}
A	area, m^2
C	effective thermal conductivity ratio
c_p	specific heat, $J \cdot kg^{-1} \cdot K^{-1}$
D	effective Biot number
Da	Darcy number
D_h	Hydraulic diameter, $D_h = 2(r_2 - r_1)$, m
d_p	Pore diameter, m
f	friction factor
h	heat transfer coefficient, $W \cdot m^{-2} \cdot K^{-1}$
h_{sf}	local convective heat transfer coefficient, $W \cdot m^{-2} \cdot K^{-1}$
K	permeability, m^2
Kn	Knudsen number
Kn_{pr}	practical Knudsen number, $Kn_{pr} = Kn/(R_2 - 1)$
k	thermal conductivity, $W \cdot m^{-1} \cdot K^{-1}$
k_d	dispersion thermal conductivity, $W \cdot m^{-1} \cdot K^{-1}$
k_r	thermal conductivity ratio ($k_r = k_f/k_s$)
M	viscosity ratio
Nu	Nusselt number
p	pressure, $N \cdot m^{-2}$
P	the dimensionless pressure drop
Pr	Prandtl number
q	surface heat flux, $W \cdot m^{-2}$
r	radius, m
r_1	inner radius, m
r_2	outer radius, m
R	dimensionless radius
R_d	relative temperature deviation
R_2	radius ratio
Re	Reynolds number
s	the inverse of a modified Darcy number
t	dimensionless factor
T	temperature, K
T_{fb}	mean temperature of the bulk fluid, K
u	velocity, $m \cdot s^{-1}$
u_m	mean velocity, $m \cdot s^{-1}$
U	dimensionless velocity
x	axial position, m

Greek symbols

β_T	thermal slip coefficient
β_v	velocity slip coefficient
γ	the ratio of specific heat ($\gamma = c_p/c_v$)
ε	porosity
θ	dimensionless temperature
μ	dynamic viscosity, $kg \cdot m^{-1} \cdot s^{-1}$
ξ	heat flux (HF) ratio
ρ	density, $kg \cdot m^{-3}$
σ_T	thermal accommodation coefficient
σ_v	tangential momentum accommodation coefficient (velocity)
φ	polar angle, rad

Subscripts

1	inner wall
2	outer wall
b	bulk
e	effective
f	fluid
fe	the effective value of the fluid
m	mean
s	solid
se	the effective value of the solid
w	wall

analytical solutions for pulsatile flow through annular, rectangular, and sector ducts filled with porous media. Xu et al. [14] performed a theoretical study on forced convection in a porous medium parallel-plate channel with the analytical, numerical, and fin-analysis methods, and the results from different methods are comprehensively compared with each other. Lee and Vafai [15] analytically studied the non-equilibrium heat transfer in a parallel-plate channel filled with porous media with symmetrical heat flux via LTNE model. Lu et al. [16] analytically modeled the fully-developed forced convection in metal foams and analyzed the effects of different parameters on overall thermal performance. Zhao et al. [17] conducted the analytical study of forced convection in porous foam annular duct and evaluated the thermal performance of tube-in-tube heat exchangers filled with highly-conductive metal foams. Xu et al. [18] performed a theoretical study on flow and heat transfer in porous foam solar collector based on Darcy/Brinkman flow model and LTNE/fin model, and compared the numerical result with the analytical result to validate the analytical solution. Yang and Vafai [19] presented the analysis on forced convection in the porous media parallel-plate channel with an internal heat source and put their focus on temperature bifurcation for two different models of wall boundary condition. Xu et al. [20] numerically investigated the forced convective heat transfer in porous foams and analyzed the five influencing factors for LTNE effect. It was found that the porosity and the thermal conductivity ratio are two main factors for the LTNE effect in porous media.

In the continuum flow, the intermolecular collisions dominate internal flow in the macro-scale duct, and the Knudsen number of this case is generally less than 10^{-3}. When the channel characteristic length is comparable to the molecular mean free path of the fluid, the effect of intermolecular collisions is almost equivalent to that of collisions between molecules and channel wall. In this case, the Knudsen number is in the range $10^{-3} < Kn < 10^{-1}$, and this is slip flow. If the channel scale is sufficiently small, the effect of intermolecular collisions can be neglected and molecule-wall collisions play a dominant role. This corresponds to the range of Knudsen number, $Kn > 10$, which is called the free molecular flow. The Knudsen number range $10^{-1} < Kn < 10$ corresponds to the transition flow.

The focus of this work is emphasized on the slip flow and the slip heat transfer in a porous medium channel. The slip phenomenon in a porous medium channel have been investigated to some extent. Shokouhmand et al. [21] performed a numerical simulation of the convective heat transfer in a microchannel filled with a porous medium with the effect of a varying Knudsen number. Hooman [22] obtained the analytical solution of forced convective heat transfer in a porous medium rectangular duct with H2 boundary condition based on LTE assumption. Hashemi et al. [23] presented an analytical solution for micro-annulus filled with a porous medium using the LTE assumption. Xu [24] presented an analytical and numerical study on thermal performance of a multi-layer porous medium heat exchanger with the effect of the