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Unstable buoyant flow in a vertical porous layer with convective boundary conditions

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ABSTRACT

The instability of natural convection in a vertical porous layer is analysed. The plane parallel boundaries of the vertical layer are modelled as open and subject to Robin-type temperature conditions. The latter conditions describe heat transfer to the external environment, with a finite conductance. The basic state is given by a stationary fully-developed flow with linear velocity and temperature profiles. Instability arises when the Darcy-Rayleigh number exceeds its critical value. This value depends on the Biot number associated with the temperature boundary conditions. The most unstable normal modes turn out to be transverse. By solving numerically the stability eigenvalue problem, it is shown that the critical Darcy-Rayleigh number is a decreasing function of the Biot number when the Biot number is sufficiently small. For larger Biot numbers, a minimum is attained, and then the critical Darcy-Rayleigh number becomes an increasing function of the Biot number.

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1. Introduction

Stationary free convection in a vertical porous layer with infinite height may take place in the form of a parallel bidirectional flow with linear velocity and temperature profiles. In the usual formulation, the boundaries of the layer are impermeable, with unequal uniform temperatures T_1 and T_2. With this setup, the stationary parallel flow has a vanishing mass flow rate, thus describing a single convective cell spread all over the infinite height of the layer. A classical paper by Gill [1] presents a rigorous proof that this stationary buoyant flow is linearly stable. A numerical evaluation of the growth rates was carried out later on by Rees [2] and by Lewis et al. [3]. The role played by local thermal non-equilibrium between the phases was investigated by Rees [4], while the effects of nonlinearity were analysed by Straughan [5] and Scott and Straughan [6]. Barletta and Alves [7] pointed out that the linear stability proved by Gill holds true also for non-Newtonian fluids subject to a power-law rheology.

In a recent paper [8], the proof of stability drawn in Gill's paper is shown to be ineffective if the velocity boundary conditions are altered by assuming the boundaries as permeable. This change in the boundary conditions turned out to induce a linear instability occurring when the Darcy-Rayleigh number, R, is larger than 197.081 [8]. The instability selects transverse modes as the preferred patterns at onset meaning that the single-cell basic flow breaks up into a two-dimensional multicellular flow. These results were further extended to the case where a pressure difference exists across the vertical porous layer [9]. The pressure difference results into a two-dimensional basic flow instead of the one-dimensional parallel flow studied both in the original Gill's proof [1] and in Barletta [8].

Third-kind, or Robin, boundary conditions were invoked by several authors as a model of convection to an external environment. Kubitschek and Weidman [10,11] tested the effects of the Biot number on the onset of Rayleigh-Bénard instability in a horizontal porous layer. These authors employed the Biot number to model boundary heat transfer due to an external forced convection regime. We mention that, formally, the same Robin conditions were used also to describe the effects of the conductance of bounding walls having a finite thickness (see, for instance, Chelghoum et al. [12]). However, Mojtabi and Rees [13] and Rees and Mojtabi [14] proved that this use of the Robin boundary conditions may be inappropriate as, strictly speaking, the Biot number would turn out to be dependent on the wave number of the perturbation.

The aim of this contribution is to further extend the scope of the analysis carried out by Barletta [8] on replacing the isothermal

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Nomenclature

B	Biot number, Eq. (5)
$\boldsymbol{e_y}$	unit vector relative to y-axis
$f(x)$	rescaled dimensionless perturbation amplitude, Eq. (24)
$F(x)$	dimensionless perturbation amplitude, Eq. (10)
g	modulus of the gravitational acceleration $[\mathrm{m/s^2}]$
$\boldsymbol{g}$	gravitational acceleration $[\mathrm{m/s^2}]$
h_e	external heat transfer coefficient $[\mathrm{W/(m^2\ K)}]$
$H(x)$	dimensionless perturbation amplitude, Eq. (10)
k	dimensionless wave number
(k_y, k_z)	dimensionless wave vector
K	permeability $[\mathrm{m^2}]$
L	porous layer thickness [m]
p	dimensionless pressure, dimensional pressure [Pa], Eq. (2)
P	Peclét number
R	Darcy-Rayleigh number, Eq. (3)
$\widehat{R}$	dimensionless parameter, Eq. (27)
S	rescaled Darcy-Rayleigh number, Eq. (13)
$\widehat{S}$	dimensionless parameter, Eq. (22)
$S_{c,\min}, \widehat{S}_{c,\min}$	minimum values of S_c, $\widehat{S}_c$
t	dimensionless time, dimensional time [s], Eq. (2)
T	dimensionless temperature, dimensional temperature [K], Eq. (2)
T_0	reference temperature [K]
T_1	external left reservoir temperature [K]
T_2	external right reservoir temperature [K]
$\boldsymbol{u}$	dimensionless velocity, (u, v, w), dimensional velocity [m/s], Eq. (2)
v_m	basic flow dimensional average velocity [m/s], Eq. (29)
v_{py}	perturbation dimensional phase velocity [m/s], Eq. (29)
$\boldsymbol{x}$	dimensionless position, (x, y, z), dimensional position [m], Eq. (2)

Greek symbols

α	average thermal diffusivity $[\mathrm{m^2/s}]$
β	thermal expansion coefficient $[\mathrm{K^{-1}}]$
γ	dimensionless parameter, $-\eta + i\,\widehat{\omega}$
ε	dimensionless perturbation parameter, Eq. (7)
η	dimensionless growth rate, Eq. (10)
θ	temperature difference ratio, Eq. (3)
μ	dynamic viscosity [Pa s]
ν	kinematic viscosity $[\mathrm{m^2/s}]$
$\xi_{1,2}$	dimensionless real parameters, Eq. (17)
σ	heat capacity ratio
χ	average thermal conductivity [W/(m K)]
ω	dimensionless angular frequency, Eq. (10)
$\widehat{\omega}$	rescaled dimensionless angular frequency, Eq. (12)

Subscripts, Superscripts

$\sim$	perturbed quantities
$'$	derivative with respect to x-coordinate
b, c	basic state, critical value
max	modes of maximum growth rate

conditions at the boundaries with third-kind (Robin) boundary conditions for the temperature. In this way, convection to the external environment is modelled through a uniform heat transfer coefficient h_e. The dimensionless counterpart of this coefficient is the Biot number, B. When the Biot number tends to infinity, the Dirichlet temperature boundary conditions assumed by Gill [1] and Barletta [8] are recovered. Also the condition of vanishing mass flow rate in the basic state is relaxed, thus allowing for a buoyancy-induced upward or downward flow. This effect will be set up by allowing the reference temperature employed to define the buoyancy force to be lower or higher than the arithmetic mean $(T_1 + T_2)/2$.

2. Mathematical model

This study is focussed on the natural convection in a vertical porous layer. The boundaries are a pair of parallel vertical planes at $x = \pm L/2$ where convection to external fluid reservoirs at given temperatures T_1 and T_2, with $T_2 > T_1$, takes place (see Fig. 1). The (x, y, z)–axes are oriented as shown in Fig. 1.

2.1. Governing equations

Fluid flow through the porous medium is modelled by invoking Darcy's law, and by assuming that the Oberbeck-Boussinesq approximation can be applied. We will consider the effect of viscous dissipation as negligible, and we will assume the absence of internal heat sources. It is further assumed that the hypothesis of interphase thermal equilibrium can be applied locally within the fluid saturated porous medium. Under these assumptions, the local mass, momentum and energy balance equations can be expressed in a dimensionless form as [15,16],

$$\nabla \cdot \boldsymbol{u} = 0, \tag{1a}$$

$$\boldsymbol{u} = -\nabla p + RT\,\boldsymbol{e}_y, \tag{1b}$$

$$\frac{\partial T}{\partial t} + \boldsymbol{u} \cdot \nabla T = \nabla^2 T, \tag{1c}$$

where R is the Darcy-Rayleigh number and $\boldsymbol{e}_y$ is the unit vector of the y–axis. A suitable rescaling of the dimensional quantities has been defined so that the corresponding dimensionless quantities are obtained,

$$\frac{1}{L}(x, y, z) \rightarrow (x, y, z), \qquad \frac{\alpha}{\sigma L^2} t \rightarrow t, \qquad \frac{K}{\mu\alpha} p \rightarrow p,$$

$$\frac{L}{\alpha}\boldsymbol{u} = \frac{L}{\alpha}(u, v, w) \rightarrow (u, v, w) = \boldsymbol{u}, \qquad \frac{T - T_0}{T_2 - T_1} \rightarrow T. \tag{2}$$

Here, we have denoted time by t, dynamic pressure by p, seepage velocity by $\boldsymbol{u} = (u, v, w)$, and temperature by T. Moreover, K is the permeability, μ is the dynamic viscosity, α is the average thermal diffusivity, σ is the ratio between the average volumetric heat capacity of the saturated porous medium and the volumetric heat capacity of the fluid, while T_0 is the reference temperature. We recall that dynamic pressure is the difference between the pressure and the hydrostatic pressure evaluated locally within the medium.

The Darcy-Rayleigh number R and the temperature difference ratio are given by

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