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Bifurcation analysis of density wave oscillations in natural circulation loop

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ABSTRACT

The present study explores the linear stability and bifurcation analysis of two-phase flow in the single heated channel for the natural circulation loop. The bifurcation analysis of this natural circulation system is limited to the detection of Hopf bifurcation, generalized Hopf bifurcation and turning point or limit point bifurcation of limit cycles. These natural circulation loops have several engineering applications and prone to thermal-hydraulic instability, which may lead to an operationally inefficient system. Stability boundary (linearly) obtained on a N_{pch} - N_{sub} plane predicts the existence of limit cycles (a nonlinear phenomenon) for these loops and gives motivation for the further study on the detection of nonlinear phenomena. The subsequent results show a novel case of stability boundary (listed as Type A in the present work) emerging for higher subcooling numbers on N_{pch} - N_{sub} parameter plane, which has not been noted in literature earlier and lies near to the Ledinegg stability boundary, which is a static instability. The characteristics of points nearby Type A stability boundary predicts that this boundary is different from the Ledinegg stability boundary. This Type A stability boundary divided by subcritical and supercritical Hopf bifurcation, which is separated by a generalized Hopf bifurcation (GH) point. Sustained oscillations in flow velocity can be observed near to the Hopf points on this boundary. These subcritical (hard and dangerous) and supercritical (soft and safe) Hopf bifurcations have been examined with time series graphs for all types (Type A-C) of stability boundaries present for the loops and occurring on the N_{pch} - N_{sub} parameter plane. The two other type of boundaries known earlier is characterized here as Type B and C, respectively, which is basically Type I and Type II density wave oscillations. The transition of stability boundaries is discussed on the N_{pch} - N_{sub} parameter plane, which shows that Type A stability boundary contains GH bifurcation coming from Type B stability boundary. The bifurcation of limit cycles originating from the Hopf points in the system has been examined and it is found that limit point bifurcation of limit cycles is occurring in the system, originating from the generalized Hopf points.

1. Introduction

Two-phase flow instabilities for natural and forced circulation loops are well studied and discussed for various engineering applications in past few decades. Due to passive heat removal mechanism, these loops are quite useful to improve safety of nuclear reactors. Hence, next generation of nuclear reactors is likely to be based on natural circulation loops (NCL) [1,2].

Boure et al. classified two-phase flow instabilities as static and dynamic instabilities [3]. Static instabilities occur for a sudden large change in flow rate of system and steady state shift to new operating condition, whereas dynamic are the ones, which occurs due to interaction between pressure, flow rate and void generation in the system etc. Reviews of two-phase flow instabilities are given in Boure et al., Ruspini et al., Durga Prasad et al., Kakac et al. [3–5].

In mathematical sense, stability of any system is defined as change in steady state due to internal or external perturbations. If system returns to its normal steady state condition after a perturbation, system is stable, otherwise system is unstable. Stability analysis can be further categorized as linear and nonlinear, linear stability is valid only near to local stability boundary and can be detected with observation of Eigenvalues. However, nonlinear effects on system cannot be detected by this analysis. For any nonlinear system, a sudden change in steady state can lead to nonlinear phenomena such as limit cycles, which can be studied only with bifurcation analysis.

Single-phase NCL have been studied by several researchers experimentally as well as numerically in the past decades. Vijayan et al. carried out nonlinear stability analysis of rectangular single-

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phase NCL for detection of limit cycles and chaos with experimental evidence [6]. Effect of geometry and operating parameters on stability analysis were done by Basu et al. [7]. Performance comparison between rectangular and toroidal was carried out in Basu et al. [8]. Several researchers performed stability analysis with supercritical CO_2 as loop fluid [9–11]. However, present work is focused on the two-phase flow NCL, in which static and dynamic instability occur depending on operating conditions.

Density wave oscillations (DWOs) are one of the most studied dynamic instabilities for thermal-hydraulic systems. The complex interaction between pressure drop, fluctuations in flow rate and void generation in the system lead to occurrence of DWOs. In simpler term, one can define DWOs as the perturbation of density because of the time delay of propagation of liquid and vapor throughout the channel. DWOs are of two types; Type-I DWO occurs at low power and low steam quality, condition on the contrary Type II DWO occur at high power and large steam quality condition. The head change is driving force for Type I DWO whereas the frictional pressure drop is the driving force for the Type II DWO.

There are two popular approaches for stability analysis of DWOs for nuclear reactors. The Laplace transform is first approach to explore linear stability characteristic, but this tool is not very useful to study bifurcation analysis. Lee and Lee implemented Laplace transforms to investigate excursive instabilities and DWOs in open channel two phase flow NCL [12]. This method is later used to investigate these oscillations in NCLs with and without neutronic [13,14]. A similar approach is used for investigating these instabilities in Advanced Heavy Water Reactor along with numerical simulations to get nonlinear dynamics of the system [15–17]. Second approach is to investigate system using reduced order models (ROMs) for these loops with a set of ordinary differential equation containing essential features of nonlinear system of nuclear reactors. This state space approach is useful to get time series graphs numerically for detection of nonlinear patterns. These ROMs are well studied for the above-mentioned nonlinear systems [18–24].

Nonlinear analysis of present ROM for NCL was limited to the detection of subcritical and supercritical Hopf points [25]. An advance analysis is done here to get clear identifications of these points. First Lyapunov coefficients (FLCs) have been calculated for Hopf points; which identifies type of bifurcation. Subcritical Hopf is hard and dangerous bifurcation as it leads to instability in linearly stable region for relatively large perturbations. Supercritical Hopf is soft and safe bifurcation as stable limit cycles co-exist in unstable region. Subcritical Hopf and supercritical Hopf points are separated by generalized Hopf (GH) point (FLC = 0), which gives rise to limit point bifurcation of cycles (LPC) curve, a nonlinear stability boundary [26–28]. Hopf bifurcation and bifurcation of limit cycles are investigated and shown in present work. Matcont bifurcation package is used here for the analysis and steady state operating condition is calculated by a code written in MATLAB® [29,30].

Linear and nonlinear time domain analysis for DWOs in Papini et al. exhibits region of Ledinegg instability and DWOs on N_{pch} - N_{sub} plane [31]. The proposed Type A stability boundary is close to the Ledinegg stability zone but does not exhibit characteristics of saddle-node bifurcation, which is a manifestation of Ledinegg stability. The original contribution of the present work is to investigate the newly proposed Type A stability boundary. The linear and nonlinear stability analysis for this stability boundary is carried out. The "nose" type stability boundary, which is mentioned in the Papini et al., and Ambrosini et al. is a Ledinegg stability [31,32]. This nose type, Ledinegg stability are discussed in work of Pandey and Singh [33]. The NCL model adopted in this work has been studied in Paul and Singh as well but the study is limited to the Type B and C stability boundary [25]. The mathematical analysis is limited to Hopf bifurcation for these Type I and Type II DWOs.

Type B and Type C stability boundaries are combination of Type I and II DWOs. Extensive analysis of these stability boundaries has been done in past, however, study of these boundaries exhibit transition between pair of Type A and Type C stability boundaries to pair of Type B stability boundary. These types of boundaries exist and transition from one to another for different set of parameters. It should be noticed that mathematical characteristics also shift from one pair of stability boundary to other pair. Subcritical and supercritical Hopf along with LPCs have been investigated here for different type of boundaries.

2. Analytical model and bifurcation

2.1. Analytical model

The analytical model developed by Paul and Singh of two-phase flow in the NCL for a single heated channel is adopted here for the bifurcation analysis and nonlinear stability analysis [25]. A schematic diagram of natural circulation loops has been shown in the Fig. 1. The down comer level (H_t) and riser length (L_r) which is shown in the Fig. 1 for two-phase natural circulation loop, which affect the dynamics of the loop and parametric study has been carried out considering these as parameters.

The complete description of the model and procedure of the converting to partial differential equations (PDEs) of mass, momentum and energy equation of the natural circulation systems to ordinary differential equations (ODEs) has been given in that work. However, the assumptions made for the model and important equations have been presented here for the general readers. The operating conditions for the advanced heavy water reactor is used with certain assumptions. The assumptions are as follows

1. The system pressure is constant.
2. Heat flux is uniformly distributed along the channel.
3. Coolant (fluid) is incompressible.
4. Subcooled boiling in the single phase is not considered.
5. $\partial p/\partial t$ term in the energy conservation equation is neglected.
6. Heat losses in the riser as well as down comer are negligible.
7. Properties of the fluid or mixture is constant inside the riser and down comer.

The dimension less mass, momentum and energy conservation in the form of partial differential equations for this heated channel

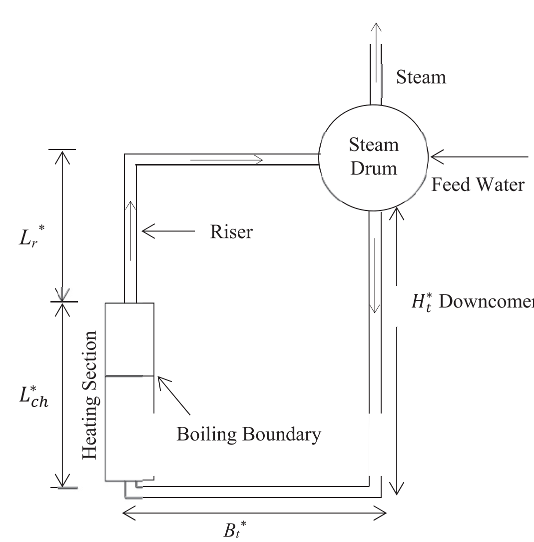

Fig. 1. Schematic of a two-phase natural circulation loop [25].