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Analytical solution of film mass-transfer on a partially wetted absorber tube

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ABSTRACT

This work presents a two-dimensional analytical solution of the governing differential equation for falling film vapour-absorption around a plain horizontal tube. The solution of the species transport equation gives the LiBr mass fraction distribution within the liquid absorptive film flowing along the tube surface and can be used to characterize the mass transfer performance of falling film absorbers or generators. By means of the inclusion of partial wetting effects at reduced solution mass flowrates, this study obtains an analytical expression of the mass transfer coefficient of these devices applicable over an extended range of operative conditions. The hypotheses of small penetration for physical absorption and constant heat flux condition are applied at the film interface to reach a closed-form solution. Fourier method is used to solve the problem and the eigenvalues obtained from the characteristic equation depend on Lewis number, Biot number and the dimensionless heat of absorption. Given the boundary condition at the wall, the two-dimensional mass fraction field of the laminar film can be expressed analytically as a function of Schmidt, Reynolds numbers, the tube dimensionless diameter and the ratio of the wetted area to the total exchange surface. Finally, mass transfer coefficient and absorbed mass flux are locally and globally investigated as functions of the influent dimensionless groups to clarify their effects on the physical process and screen the potentiality of the model. Results show notable qualitative and quantitative agreement with previous numerical solutions and experimental results from previous literature. This model constitutes a widely applicable and time-saving tool for actual system simulations, design and control.

1. Introduction

Vapour absorption cycles for heat driven energy conversion systems are increasingly employed in numerous applications [1–5] and provide a number of possibilities for low-grade energy recovery [6–9], besides the effective utilization of renewable sources [10,11]. With regards to these systems, it is well-recognized that the absorber performance has the most decisive influence on the overall efficiency, capacity and dimension, as well as the system cost [12]. Particularly, as a component of a refrigeration machine, the absorber determines the amount of refrigerant that can be steadily circulated within the operative cycle. Therefore, the higher the amount of refrigerant that can be absorbed per unit heat extracted at the absorber [13,14] is, the higher the specific cooling capacity at the evaporator will be. In the conventional case of a horizontal tube falling film heat exchanger, high transfer coefficients can be obtained without any significant pressure drop. However, the attempt to either experimentally or analytically describe and predict the complex phenomena of momentum, energy and mass transfer occurring inside this device, has not led to conclusive approaches. Accurate methods to predict heat and mass transfer performance of the absorber are critical for a global system optimization and its efficient control. Given the increasing availability of computational resources, the solution of the fundamental problems representing the thermo-physical essence of these systems has recently been directed towards numerical methods. References [15,16] presented simplified models for falling film absorption of water vapour over a horizontal tube using Nusselt's boundary layer assumptions for the momentum equation. Reference [17] studied simultaneous heat and mass transfer of an absorbing or desorbing laminar liquid film flowing over a vertical isothermal plate, considering effects such as the change in

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Nomenclature

A	Series coefficient
a	Thermal diffusivity [$m^2 \cdot s^{-1}$]
B	Integration constant
Bi	Biot Number, $Bi = \alpha_{if} L_c \cdot k^{-1}$
C	Integration constant
c	Constant group of parameters
c_p	Isobaric specific heat [$J \cdot kg^{-1} K^{-1}$]
D	Mass diffusivity [$m^2 \cdot s^{-1}$]
d	Dimensionless diameter, $d = 2r \cdot L_c^{-1}$
E	Separate variable function
g	Gravity [$m \cdot s^{-2}$]
Ga	Galileo Number, $Ga = \rho\sigma^3 \cdot \mu^{-4} g^{-1}$
G_v	Mass flux [$kg \cdot m^{-2} s^{-1}$]
h	Film thickness [m]
H	Separate variable function
i	Specific enthalpy [$kJ \cdot kg^{-1}$]
k	Thermal conductivity [$W \cdot m^{-1} K^{-1}$]
l_{abs}	Heat of absorption [$kJ \cdot kg^{-1}$]
Lc	Characteristic length [m], $L_c = \nu^{2/3} g^{-1/3}$
Le	Lewis Number, $Le = a \cdot D^{-1}$
mtc	Mass transfer coefficient [$m \cdot s^{-1}$]
p	Vapour pressure [kPa]
r	Outer tube radius [m]
Re	Reynolds Number, $Re = \Gamma \cdot \mu^{-1}$
Sc	Schmidt Number, $Sc = \mu \cdot \rho^{-1} D^{-1}$
Sh	Sherwood Number, $Sh = mtc \cdot L_c \cdot D^{-1}$
T	Temperature [K]
u	Streamwise velocity [$m \cdot s^{-1}$]
v	Normal velocity [$m \cdot s^{-1}$]
x	Local tangential position [m]
X	Dimensionless mass fraction, $X = \omega \cdot \omega_{in}^{-1}$
y	Local normal position [m]
w	Wetted part of the reference axial length of the tube [m]
W	Reference axial length of the tube [m]
WR	Wetting Ratio, $WR = w \cdot W^{-1}$

Greek symbols

α	Heat transfer coefficient [$W \cdot m^{-2} K^{-1}$]
β	Angular position, $\beta = x \cdot r^{-1}$
ΔT	Temperature difference, $\Delta T = T_{if}$-T_b [K]
ε	Dimensionless tangential position, $\varepsilon = x \cdot \pi^{-1} r^{-1}$
η	Dimensionless normal position, $\eta = y \cdot h^{-1}$
λ	Variable-separation constant
Λ	Normalized heat of absorption, $\Lambda = l_{abs} \cdot \Delta T^{-1} c_p^{-1}$
Γ	Mass flowrate per unit length [$kg \cdot s^{-1} m^{-1}$]
δ	Dimensionless film thickness, $\delta = h\rho^{3/5} g^{2/5} \cdot 15^{-1/5} \mu^{-2/5} \sigma^{-1/5}$
μ	Viscosity [$Pa \cdot s$]
ρ	Density [$kg \cdot m^{-3}$]
σ	Surface tension [$J \cdot m^{-2}$]
ω	LiBr mass fraction
θ	Contact angle [^{o}]

Subscripts

b	Bulk value
e	Thermodynamic equilibrium
if	Interface
in	Inlet
max	Maximum
n,m,k	Series index
w	Wall
0	Film breaking condition

properties and differential heat of solution due to inter-diffusion, using a homogeneous average velocity distribution and constant film thickness as simplifying assumptions. However, in order to capture the physics of the problem, properly formulated analytical approaches still maintain their fundamental importance as adequately accurate time-saving methods. Previous analytical solutions have been extracted while overlooking partial wetting phenomena at reduced Reynolds operability by assuming entirely active transfer interfaces and limiting the geometry of the problem to a corresponding vertical plain-wall, under the assumptions of uniform velocity profile and film thickness [18–20]. According to [19], this assumption is responsible for deviations of approximately 20% in the heat and mass transfer coefficient. To extend the applicability of analytical solutions in this field, this work obtains a closed solution for vapour absorption in a laminar, gravity driven film, flowing over a horizontal tube with circular geometry, taking into account the effect of the incomplete wetting of the tube surface, when inlet conditions can be arbitrarily selected.

2. Physical model

The system under consideration is schematically illustrated in Fig. 1. A single horizontal tube is considered to be in a saturated vapour environment. A thin film of LiBr-H_2O solution impinges at the top of the tube ($x = 0$) and flows viscously, driven by gravity, as a laminar incompressible liquid. Meanwhile, vapour absorption occurs at the free interface.

The effects of the circular cross-section of the tube, the local distributions of the film thickness and the surrounding velocity field, as well as the partial wetting of the tube surface, are taken into account for generally selectable inlet conditions. The resulting mass transfer characteristics have been considered under the following main assumptions: the zone of impingement is supposed to be a small fraction of the total periphery; the flow is laminar and there are no interfacial waves [21]; there is no shear force between the liquid film and the vapour; thermo-physical properties of the absorptive solution are supposed to not incur into significant local variation along the cross-section of a single tube; interfacial mass transfer resistance is ignored; the variation of the mass flowrate due to vapour absorption is considered to be negligible and, according to the thin film approximation [22], body fitted coordinates (x along the tube surface and y normally defined at each point) are used since the film thickness remains small if compared to the tube diameter.

2.1. Differential momentum balance

Neglecting unsteady and convective terms, pressure losses and diffusive terms in the radial direction, the momentum equation for the falling film of solution is represented by eq. (1).

$$\mu \frac{\partial^2 u}{\partial y^2} = -\rho g \sin \beta \tag{1}$$

This is solved under the hypotheses that yield to Nusselt integral solution for a laminar film over a horizontal tube (eq. (2)).