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Application of iterative algorithms for gas-turbine blades cooling optimization

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ABSTRACT

This paper presents two iterative algorithms used to solve the inverse problem of gas-turbine blade cooling. First of them was obtained by variational calculus methods, and the second one is reduced to solving the least-square approximation problem. To determine the temperature distribution on the inner boundary of the blade (of the multiply-connected region), a combination of trigonometric functions defined on the boundary of a unit circle was used. Both algorithms are presented in two variants: the first one is about reducing the number of trigonometric functions, and in the second one, coefficients of trigonometric functions are regularized in the process of iteration. Results of calculations indicate that both algorithms operate more effectively in the variant where coefficients of linear combination of trigonometric functions are regularized.

1. Introduction

Problems related to cooling gas-turbine blades are of crucial importance in power industry as they concern with increasing the efficiency of turbines, among other things. Temperature of gas flowing around the blades in the turbine influences significantly on the efficiency of the turbine, which grows with the increase of the gas temperature. A major limitation influencing on the increase of the gas temperature is a material the blade is made of. Increase of the gas temperature above a certain value results in loss of strength properties of the material what in turn leads to a damage to the turbine. To avoid it, protective coatings or blade cooling systems are used, what allows the gas temperature to increase significantly and not to cause any damage to the turbine blades. Trends related to application of cooling channels and protective coatings to the gas-turbine blades are described in papers [1]. Problems related to particular solutions of cooling system comprise, among other things, minimization of thermal stresses as well as the shape and arrangement of cooling channels inside the blade. Problems of optimization of the gas-turbine blades cooling process were discussed in papers [4–8].

Problems of gas-turbine blades cooling optimization are not typical boundary problems since the boundary condition on the walls of the cooling channels is not known, and in some problems the shape of the channel is not defined. Problems of that type belong to the class of inverse problems ill-posed in the Hadamard sense [2]. According to the classification of inverse problems presented in paper [3], inverse problems in which there is no boundary condition on the walls of the cooling channels belong to a group of problems related to identification of boundary conditions, and problems in which the shape of cooling channels is not defined belong to geometric inverse problems.

Methods of solving inverse problems are widely discussed in literature, just to mention papers [9–13]. An inverse problem being the subject matter of this paper is the Cauchy problem for the Laplace's equation. Presented algorithms for solving problems of that type are based on some mechanisms for improving the solution to the inverse problem, just as it is in the case of the SVD (singular value decomposition) algorithm [11] and regularization [9,12].

Cauchy-type inverse problems are known and solved using various numerical methods. Papers [14,15] present variational methods for solving problems of that type. Regularization of the Cauchy problem is discussed in papers [16–18], just to mention a few, and iterative algorithms are presented in papers [19–21], among others.

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2. Iterative algorithms for determining temperature distribution

In the multiply-connected region Ω, Fig. 1, the following heat conduction equation is given

$$\Delta T = 0 \tag{1}$$

with boundary conditions of the first and third types on the outer boundary of the multiply-connected region

$$\Gamma_{out}: \quad \begin{aligned} & -k\frac{\partial T}{\partial n} = h\left(T - T_f\right) \\ & T = T_o \end{aligned} \tag{2a}$$

where k [W/mK] is the heat conduction coefficient, h [W/m^{2}K] is the heat transfer coefficient, T_f [K] is the temperature of the fluid surrounding the region Ω from the outside, and T_o [K] is the temperature on the Γ_{out} boundary. Distribution of temperature and the normal derivative of temperature on the Γ_{in} boundary of the multiply-connected region Ω should be found (the Cauchy problem).

Simultaneous fulfilment of conditions (2a) is difficult, therefore the condition of the third type will be fulfilled exactly, and the condition of the first type will be fulfilled in the least square sense as the minimum of the functional

$$\begin{aligned} & \Gamma_{out}: \quad -k\frac{\partial T}{\partial n} = h\left(T - T_f\right) \\ & \min_T \int_{\Gamma_{out}} (T - T_o)^2 ds \end{aligned} \tag{2b}$$

Temperature on the Γ_{in} boundary is sought in the form

$$\Gamma_{in}: \quad T = \sum_{i=0}^{n} c_i \varphi_i \tag{3}$$

where basis functions ϕ_i form a complete system of functions.

Solution of the equation (1) with the boundary condition of the third type (2) on the Γ_{out} boundary and the boundary condition of the first type (3) on the Γ_{in} boundary may be approximated using the linear combination

$$\Omega: \quad T = T_1 + \sum_{i=0}^{n} c_i \psi_i \tag{4}$$

where the function T_1 is the solution of the equation (1) with boundary conditions:

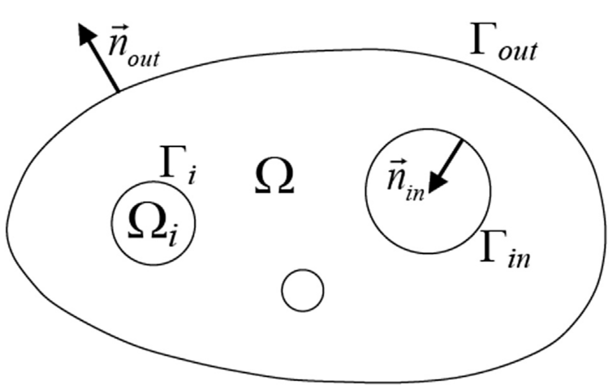

Fig. 1. Multiply-connected region Ω.

$$\begin{aligned} & \Gamma_{out}: \quad -k\frac{\partial T_1}{\partial n} = h\left(T_1 + \sum_{i=0}^{n} c_i \psi_i - T_f\right) \\ & \Gamma_{in}: \quad T_1 = 0 \end{aligned} \tag{5}$$

and basis functions ψ_i (Trefftz functions, [22]) satisfy the equation (1) with boundary conditions

$$\begin{aligned} & \Gamma_{out}: \quad \frac{\partial \psi_i}{\partial n} = 0 \\ & \Gamma_{in}: \quad \psi_i = \varphi_i \end{aligned} \tag{6}$$

It may be verified that the function given by the formula (4) satisfies the equation (1) with the boundary condition of the third type on the Γ_{out} boundary (2) and with the boundary condition of the first type (3) on the Γ_{in} boundary. To do so, it is enough to multiply boundary conditions (6) by coefficients c_i, and next to total up and add to boundary conditions (5)

$$\begin{aligned} & \Gamma_{out}: \quad -k\left(\frac{\partial T_1}{\partial n} + \sum_{i=0}^{n} c_i \frac{\partial \psi_i}{\partial n}\right) = h\left(T_1 + \sum_{i=0}^{n} c_i \psi_i - T_f\right) \\ & \Gamma_{in}: \quad T_1 + \sum_{i=0}^{n} c_i \psi_i = \sum_{i=0}^{n} c_i \varphi_i \end{aligned}$$

Unknown coefficients c_i will be determined from the minimum of functional expressing the root-mean-square distance of the solution in the form (4) on the Γ_{out} boundary from the set temperature T_o

$$J[c_i] = \frac{1}{2} \int_{\Gamma_{out}} \left(T_1(c_i) + \sum_{i=0}^{n} c_i \psi_i - T_o\right)^2 ds \tag{7}$$

2.1. Algorithm with variational method

Functional (7) reaches its minimum when its variation according to coefficients c_i

$$\delta J[c_i] = \int_{\Gamma_{out}} \left(T_1(c_i) + \sum_{i=0}^{n} c_i \psi_i - T_o\right)\left(\delta T_1 + \sum_{i=0}^{n} \delta c_i \psi_i\right) ds \tag{8}$$

is equal to zero. Variation of the function T_1 within the region Ω and on the Γ_{out} boundary will be determined from formulae (1) and (5):

$$\begin{aligned} & \Omega: \quad \Delta \delta T_1 = 0 \\ & \Gamma_{out}: \quad -k\frac{\partial \delta T_1}{\partial n} = h\left(\delta T_1 + \sum_{i=0}^{n} \delta c_i \psi_i\right) \\ & \Gamma_{in}: \quad \delta T_1 = 0 \end{aligned} \tag{9}$$

Multiplying the equation (1) by the function p, and then integrating in the region Ω as well as using the Green's identity one obtain after some transformations:

$$\int_{\Omega} k(p\Delta\delta T_1 - \delta T_1 \Delta p) d\omega = \int_{\Gamma_{out} \cup \Gamma_{in}} \left(pk\frac{\partial \delta T_1}{\partial n} - \delta T_1 k\frac{\partial p}{\partial n}\right) ds \tag{10}$$

Identity (10) including (9) is as follows: