International Journal of Thermal Sciences 118 (2017) 292–302

Contents lists available at ScienceDirect

International Journal of Thermal Sciences

journal homepage: www.elsevier.com/locate/ijts

Analytical heat conduction model of a composite material based on complete spatial randomness of filler in base matrix

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ARTICLE INFO

Article history:
Received 23 November 2016
Received in revised form
19 April 2017
Accepted 20 April 2017

Keywords:
Composite materials
Spatial randomness
Poisson's distribution
Thermal conductivity
Filler materials
Base matrix
Analytical modeling

ABSTRACT

In this work, we propose a novel analytical model to obtain the thermal conductivity of a composite material comprising of filler particles randomly distributed in a base matrix. The model assumes complete spatial randomness of the filler particles in the matrix and this can be described by Poisson's distribution. This concept has been applied to arrive at the thermal conductivity of a composite material with Aluminium powder as the filler in the Epoxy matrix. The results obtained by the proposed model have been compared with the experimental results as well as with the other established models such as Maxwell model, Bruggeman model, Lewis - Nielsen model, Russell model and Rule of Mixtures. This comparison reveals that the proposed model predicts the experimental results more accurately than the others. Also the proposed model is able to predict the experimental results of thermal conductivity of Aluminium-Epoxy composite over the entire temperature range from 4.2 K to 300 K. When applied to other published experimental data of similar composites, the model is found to predict the results fairly well with minimal aberrations. Due to the non-empirical nature of the proposed model, perhaps it may be useful for the prediction of other properties of composite materials involving the filler in a base matrix.

1. Introduction

Thermal conductivity (TC) is an important basic property of materials. It is essential to know the precise value of this property, based on which the suitable material for a specific application can be chosen. This is important for the thermal design of individual components as well as the total system. Thermal conductivity of a material depends on various parameters such as temperature, composition, impurities and structure of the material etc. With the advancement of technology, more and more composite materials are being developed whose thermal conductivities are yet to be known. From the engineering point of view, the knowledge of thermal conductivity of new composite materials is quite important for its applications.

This article deals with the type of composite materials where filler particles are randomly distributed in a base matrix. A heat conduction model has been developed which is able to predict the thermal conductivity of composite material with the known volume fraction of filler material from the thermal conductivities of parent materials more accurately compared to some established models. The above model has been applied to Aluminium-Epoxy matrix composite and the TC values predicted by the proposed model are in good agreement with the experimental data, in the temperature range from 4.2 K to 300 K.

1.1. Models used for predicting TC of composites

Several models have been developed to determine the thermal conductivity of composite materials [1–9]. Some of them are given below. Maxwell [1] assumed random dispersion of non-interacting homogeneous spherical filler particles of thermal conductivity k_f within a continuous substrate of thermal conductivity k_m to obtain the thermal conductivity of the system. However it holds good only for low volume fraction of filler material because it assumes no thermal interaction between the particles.

The effective thermal conductivity k_{eff} is given by:

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http://dx.doi.org/10.1016/j.ijthermalsci.2017.04.018
1290-0729/

Nomenclature

Abbreviations

TC	Thermal Conductivity

Symbols used

A	Cross section area, m^2
k_{eff}	Effective TC, Wm^{-1}K^{-1}
k_f	TC of filler material, Wm^{-1}K^{-1}
k_m	TC of base matrix, Wm^{-1}K^{-1}
k_{th}	Theoretical TC, Wm^{-1}K^{-1}
k_{exp}	Experimental TC, Wm^{-1}K^{-1}
Φ	Filler volume fraction, dimensionless
Φc	Critical filler volume fraction, dimensionless
l	Length of the sample, m
l^*	Modified length of the sample, m
r_p	Average radius of the filler particle, m
h_p	Average height of the filler particle, m
V	Volume of the cuboidal sample, m^3
v	Volume of single Aluminium or Epoxy cylinder, m^3
N_T	Total number of cylinders, dimensionless
N_{Al}	Total number of Aluminium cylinders, dimensionless
N_{Ep}	Total number of Epoxy cylinders, dimensionless
N_{row}	Number of rows, dimensionless
N_{col}	Number of columns, dimensionless
λ	Mean number of particles, dimensionless
X	Random variable, dimensionless
k	Integer value in a certain range, dimensionless
k_i	Effective thermal conductivity of i^{th} column, Wm^{-1}K^{-1}
$\boldsymbol{W_i}$	i^{th} component of $\boldsymbol{W}$ vector, dimensionless
$\boldsymbol{P_i}$	i^{th} component of probability vector, dimensionless

$$\frac{k_{eff}}{k_m} = \left(\frac{k_f + 2k_m + 2\Phi\left(k_f - k_m\right)}{k_f + 2k_m - \Phi\left(k_f - km\right)}\right) \tag{1}$$

Bruggeman's model [2] allows predicting many properties of composite materials such as thermal conductivity, thermal diffusivity, magnetic permeability etc. This theory assumes that a composite material may be constructed incrementally by introducing infinitesimal changes to an already existing material. This approach is known as Differential Effective Medium theory (DEM). In this model, k_{eff} is given as,

$$\frac{k_{eff} - k_f}{\left(k_{eff}\right)^{1/3}} = \frac{(1-\Phi)\left(k_m - k_f\right)}{(k_m)^{1/3}} \tag{2}$$

Lewis-Nielsen [3] proposed another empirical model which is applicable upto the volume fraction Φ ~40 % of filler in base matrix. The effective thermal conductivity k_{eff} is given as,

$$k_{eff} = k_m\left(\frac{1 + PQ\Phi}{1 - P\Phi\Psi}\right) \tag{3}$$

where P, Q and Ψ are constants depending on the shape of the filler particles. Russell's theoretical model [4] calculates the thermal conductivity according to the formula:

$$k_{eff} = k_m\left(\frac{\Phi^{\frac{2}{3}} + \frac{k_f}{k_m}\left(1 - \Phi^{\frac{2}{3}}\right)}{\left(\Phi^{\frac{2}{3}} - \Phi + \frac{k_f}{k_m}\left(1 + \Phi - \Phi^{\frac{2}{3}}\right)\right)}\right) \tag{4}$$

The well-known Rule of mixtures [5] is another model which is used by several researchers for determination of thermal conductivity in composites. k_{eff} for series and parallel configurations are given by equations (5a) and (5b) respectively.

$$k_{eff} = (1-\Phi)k_m + \Phi k_f \tag{5a}$$

$$\frac{1}{k_{eff}} = \frac{(1-\Phi)}{k_m} + \frac{\Phi}{k_f} \tag{5b}$$

A few related works in this field paved the way of the proposed analytical model. Presently available statistical and mathematical models [10–15] for estimation of TC of composites and polymers imparted crucial understanding in the development of the proposed model. Results from available numerical heat transfer models [16–19] of composites and experimental work [20–27] especially at low temperatures aided in foreseeing the trend of TC curve as a function of volume fraction as well as temperature.

2. Development of a new analytical model

In spite of the availability of several models to predict the TC, most of them are empirical and lack the physical understanding of the heat transfer mechanisms. Hence we propose here, a new analytical model which leads to a better visualization and understanding of the heat transfer through the composite and predicts the TC more accurately than the other models in the temperature range from 4.2 K to 300 K.

2.1. Implementation of the model to a specific composite material

Although, the model can be implemented for any composite material containing the filler particles suspended randomly in a substrate, we will consider the specific example of Aluminium-Epoxy composite. Here, the filler material is Aluminium powder and the matrix is the Epoxy. The thermal conductivity values at 300 K for Aluminium and Epoxy are ~237 $W\,m^{-1}\,K^{-1}$ [28–30] and ~1.052 $W\,m^{-1}\,K^{-1}$ [31] respectively.

For the implementation of the analytical model, the dimensions of the filler particles (Aluminium, in the present case) are required. Although the filler particles may have irregular shapes, they can be approximated to be either cylinders or rectangular cuboids. SEM analysis can be used for the determination of the average width and height of the particles. Fig. 1(a) and (b) show the identical SEM photographs. Fig. 1(a) and (b) are used for the measurements of the average diameter and the height of the Aluminium particles (assumed as cylinders) respectively. The dimensions of nearly 100 particles have been measured to arrive at the average diameter and height of the particle. By the above measurements, the average values for the radius r_p and height h_p have been obtained as ~4.5 μm and ~19 μm respectively. For the sake of simplicity, it is assumed the Epoxy particle also has similar dimensions as that of the Aluminium particle.

Since the Aluminium-Epoxy composite can be assumed to be isotropic, for simplicity, we propose a one dimensional (1D) model