International Journal of Thermal Sciences 118 (2017) 410–422

Contents lists available at ScienceDirect

International Journal of Thermal Sciences

journal homepage: www.elsevier.com/locate/ijts

Numerical simulation of unsteady squeezing nanofluid and heat flow between two parallel plates using wavelets

R.C. Mittal, Sapna Pandit*

Department of Mathematics, Indian Institute of Technology Roorkee, Roorkee 247667, India

A R T I C L E   I N F O

Article history:
Received 25 October 2016
Received in revised form
20 April 2017
Accepted 21 April 2017

Keywords:
Squeezing flow
Cu-water nano fluid
Quasilinearization technique
Scale-3 Haar wavelets
Collocation points

A B S T R A C T

In the present study, unsteady squeezing nano fluid flow between two parallel plates has been investigated first time with wavelets. Similarity transformation has been used to transform the governing nonlinear momentum and thermal energy equation to a nonlinear coupled ordinary differential equation with fitted boundary conditions. For nonlinear coupled ordinary differential equation we present the new perspective of Haar wavelet transform with essential idea of scale-3 Haar wavelets. The numerical investigation is carried out for various physical parameters such as squeeze number, Reynolds number, Eckert number and nano particle volume fraction. As a main outcome from the present study is that, wavelet yields more accurate results than the other numerical results that are available in the literature. We have used Haar wavelets with scale-2 and scale-3 parameters for dilation and it has been found that wavelets with scale-3 produces better results than scale-2 wavelets.

1. Introduction

Nano fluids are widely encountered in many industrial and technology applications since it has the potential to increase the heat transfer rate. Recently research on heat and mass transfer for unsteady squeezing viscous flow between two parallel plates has recently stimulated the interest of researchers and engineers. Heat transfers by natural convection have many industrial and engineering applications such as hydro dynamical machines, polymer processing, lubrication system, chemical processing equipment, formation and dispersion of fog, damage of crops due to freezing, food processing and cooling towers, gas turbine engine, geo thermal system, heat exchangers and nuclear waste repositories etc. Some fluids like water, ethylene glycol and engine oil have poor heat transfer properties where metal heat conductivity is three time higher than these fluids. Thus combinations of these two substances generates the thermal conductivity as like metal and produce the flow of heat transfer medium same as fluid. Such flows are especial interest in applications to bearings with liquid-metal lubrication.

Squeezing flow between two parallel disks has been active area of research because of its occurrence in engineering applications. In order to enhance heat transfer rate in nanofluids but in planned way some researchers has been study it analytically and numerically. Analytical analysis using Homotopy perturbation method for magneto-hydro-dynamic squeezing flow of a viscous fluid between parallel disks has been proposed by Domairry et al. [1]. Another analytical investigation has been done using Differential transform method as well as by Runge Kutta-4 method with shooting technique [2] for Squeezing flow of Cu-water and Cu-kerosene nanofluids between two parallel plates. The heat transfer enhancement in two dimensional enclosure utilizing nanofluids has been investigated numerically using finite volume approach with direct implicit scheme by Khanafer et al. [3]. Finally the conclusion is that the suspended nano-particles increase the heat transfer rate at any given Grashof number. Sheikholeslami et al. [4,5] investigate the effect of a magnetic on a natural convection in an incline half annulus enclosure filled with Cu-water nano-fluids using control volume based finite element method and Lattice Boltzmann method. The conclusion of the investigation of [4] and [5] are; Hartmann number and the inclination angle of the enclosure can be controlled parameters at different Rayleigh number and heat transfer increases as Hartmann number but it decreases with Rayleigh number. More recent work on the subject numerical studies on nano-fluid flow and heat and mass transfer can be seen in Refs. [7–9,19,21–25].

* Corresponding author.
E-mail addresses: mittalrc@gmail.com (R.C. Mittal), sappu15maths@gmail.com (S. Pandit).

http://dx.doi.org/10.1016/j.ijthermalsci.2017.04.019

Nomenclature

HWM	Haar wavelet method
Num	Numerical method
u	Velocity component in x direction
v	velocity component in y direction
h	suspension height
E_c	Eckert number
P_r	Prandtl number
ρ	fluid density
ϕ	nanoparticle volume fraction
g	dimensionless temperature
$(\rho C_p)_{nf}$	effective heat capacity
ρ_{nf}	effective density
μ_{nf}	effective dynamic viscosity
k_{nf}	effective thermal conductivity

Recently, engineers and scientist have been increasing interest to analyze the solutions to different classes of partial and ordinary differential equations due to the fact that they have numerous applications in natural science and engineering. Analytic or numerical methods are essential to solve this wide class of differential equations. Some time analytic method fails to tackle the nonlinearity, detect the singularity and to derive the precise analytical solution, in such cases numerical methods are fundamentally important.

In the recent years wavelet approach is becoming more popular in the field of numerical approximations. Wavelet methods have been used for abrupt change detection. The present nanofluid flow problem has been affected with various parameters such as squeeze number, Reynolds number, Eckert number and nano particle volume fraction. Thus, we have used Haar wavelet method for the present problem. Haar wavelet method is much easier to implement as compare to Legendre and Chebyshev wavelets. Its computation cost is almost negligible and its order of convergence increases as we shift Haar function from scale-2 to scale-3. The present approach is applicable on both BVPs and IVPs with a modification in the basis of Haar wavelets.

A short introduction to the scale-2 Haar wavelets and its applications can be found in Ref. [10]. Motivated by excellent performance of scale-2 Haar wavelets [10–13,15], we extend this idea with some modification for development of scale-3 Haar wavelets. Compactly supported scale-3 Haar wavelet [14] is the modified version of scale-2 Haar wavelets. In scale-3 Haar wavelets there exists a scaling function $\varphi(x)$ and two wavelets; one is symmetric $\psi^1(x)$ and other is anti-symmetric $\psi^2(x)$. The most interesting and important property of scale-3 Haar wavelets is, its resolution level increases in multiples of three which is more better then scale-2, since increment in scale of wavelets increase the resolution quickly. Another useful property for a mathematician is that through wavelets we can compute the solutions of any kind of partial and ordinary differential equation easily and at any point in the domain. Researchers are attempting to find more accurate theory to predict the changes of behavior with nanofluids thus investigation using wavelets analysis is best suitable to study the nanofluids. From Table 3 and Table 4 we can predict that our novel approach of scale-3 Haar wavelets predict much better results as compare to available other numerical method and scale-2 Haar wavelets.

2. Problem development

Let us consider an unsteady two dimensional squeezing nanofluid flow between two infinite parallel plates. The distance between these two parallel plates is $z = \pm H(1-\alpha t)^{1/2} = \pm h(t)$. H is the initial position at time $t = 0$. They are embedded in a medium filled with based nanofluids containing nano particles. The base fluids and nano particles are assumed to be in thermal equilibrium without any slip between base fluids and nanoparticles. Here α refers as squeezed parameter which is inversely proportional to the time. These two plates are squeezed to each other for $\alpha > 0$ and separated to each other for $\alpha < 0$. Also, viscous dissipation effects are maintained to analyze the generation of heat due to friction caused by shear in the flow. The importance of this effect can be visualized for high Eckert number since in this case the fluid is highly viscous or flowing with high speed. In present analysis nanofluids is the combination of water and copper. The characteristics of this component mixture are; incompressible, no chemical reaction, negligible viscous dissipation and negligible radiative heat transfer etc. The governing equations for conservative momentum and energy in unsteady two dimensional flow of a nanofluid fluid from the geometry of Fig. 1 as given below

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\rho_{nf}\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu_{nf}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$\rho_{nf}\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial y} + \mu_{nf}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \tag{3}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k_{nf}}{(\rho C_p)_{nf}}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \frac{\mu_{nf}}{(\rho C_p)_{nf}}\left(4\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2\right) \tag{4}$$

with the boundary conditions as follows

$$\begin{aligned} &u = 0, v = \frac{dh}{dt}, T = T_H \quad \text{at} \quad y = h(t) \\ &u = 0, v = 0, \frac{\partial T}{\partial y} = 0 \quad \text{at} \quad y = 0 \end{aligned} \tag{5}$$

where u, v are the velocity components along x and y axes, T

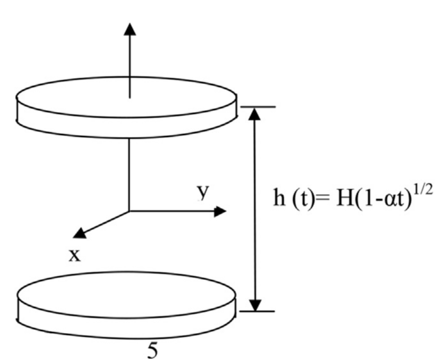

Fig. 1. Geometry of physical model.