International Journal of Thermal Sciences 118 (2017) 461–474

Contents lists available at ScienceDirect

International Journal of Thermal Sciences

journal homepage: www.elsevier.com/locate/ijts

Thermal spreading analysis of a transversely isotropic heat spreader

Juhyeong Lee [a], Scott M. Thompson [b, *], Thomas E. Lacy Jr. [a]

[a] Department of Aerospace Engineering, Mississippi State University, Mississippi State, MS, 39762, United States
[b] Laboratory for Fatigue & Additive Manufacturing Excellence (FAME), Department of Mechanical Engineering, Auburn University, Auburn, AL, 36849, United States

ARTICLE INFO

Article history:
Received 19 September 2016
Received in revised form
11 May 2017
Accepted 11 May 2017

Keywords:
Transversely isotropic
Composite material
Multi-phase heat spreader
High heat flux
Conduction

ABSTRACT

An analytical solution for the steady-state temperature distribution in a transversely isotropic (TI) heat spreader is provided and validated using three-dimensional finite element (FE) analysis. The dimensionless maximum temperature and corresponding thermal spreading resistance were determined for various Biot numbers, dimensionless heat spreader thicknesses, source-to-spreader area ratios, and thermal conductivity ratios (ratio of out-of-plane to in-plane thermal conductivities). The heat spreader investigated consists of uniformly-distributed fibers/channels aligned in the heat spreader's thickness direction. Solutions are presented graphically for various geometric, material, and operating mode combinations. The analytical solutions differ by less than 1% from the FE solutions, indicating that the analytical solution, with cosine solution form, is both effective and accurate in predicting the thermal spreading resistance of a TI heat spreader for many parameter combinations. These results can aid the design or analysis of non-traditional media for thermal spreading, including polymer composites, metal matrix composites, nanocomposites, heat pipes and electronics packaging materials with uniformly-distributed thermal vias.

1. Introduction

Composite materials are attractive to many industries due to their tailorable properties for a variety of design constraints. Composites generally consist of least one *reinforcement* phase surrounded by a binder or *matrix* phase. The *reinforcement* is typically stiffer, stronger, or more conductive than the *matrix* phase. The *matrix* holds each *reinforcement* in an orderly pattern while retaining its intrinsic properties [1]. Reinforcements (short and long fibers, particles, etc.) may be integrated into ceramic, metal, or polymer matrices for achieving more desirable bulk properties than that of the matrix material alone. A carbon fiber reinforced polymer (CFRP) composite is a common type of composite. Relative to traditional, single-phase metals, CFRP composites can be more lightweight, corrosion resistant, tailorable, and formable to complex shapes [2]. For these reasons, CFRP composites have been used as construction materials for assembling critical aerospace components such as airframes, fuselages, central wing boxes, and other wing components (e.g., skins, stringers/ribs, and ailerons) [3,4].

Heat spreaders are single-phase or multi-phase media used for diffusing concentrated heat fluxes, from sources such as central processing units (CPUs) or light emitting diodes (LEDs), to a heat sink for subsequent convection and/or radiation with surroundings. They are used to manage temperature, heat transfer rates, temperature gradients and interfacial thermal stresses in a variety of applications in which the performance, reliability, and safety of a heat dissipating source are of interest. Heat spreaders have been used with success for avionics thermal management, in which heat fluxes are relatively high and the assembly volume is constrained [5–11]. Single-phase heat spreaders are fabricated using a single, solid material and are typically metallic (copper, aluminum, etc.). Multi-phase heat spreaders can consist of (i) composites with multiple solid constituents or (ii) a solid encapsulating liquid and vapor, e.g., heat pipes or “thermal ground planes” [12]. During heat pipe operation, encapsulated liquid and vapor repeatedly evaporates and condenses, respectively, allowing for enhanced heat transfer. An ideal heat spreader possesses minimal volume and a near-isothermal temperature distribution (i.e., reduced temperature gradients) at its heat sink interface during operation. Multi-phase heat spreaders can be designed to have high in-plane thermal conductivity for promoting heat transfer along the heated and/or cooled surface [13,14]. In general, composites can be designed to

---

* Corresponding author. Auburn University, Mechanical Engineering, 3452 Wiggins Hall, Auburn, AL 36849, United States.
E-mail address: scott.thompson@auburn.edu (S.M. Thompson).

http://dx.doi.org/10.1016/j.ijthermalsci.2017.05.009
1290-0729/

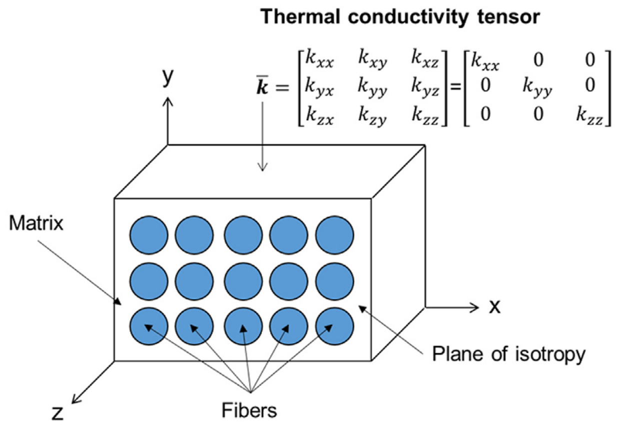
possess preferentially arranged thermally conductive reinforcements in order to achieve heat transfer in a specific direction [15,16]. In this work, conductive fibers are arranged to provide enhanced through-thickness conductivity.

Composites can possess anisotropic (i.e., directionally dependent) thermal conductivities represented by the second-order thermal conductivity tensor, $\overline{\boldsymbol{k}}$. This tensor for an anisotropic composite depends on matrix and reinforcement thermal conductivities, reinforcement orientation distribution, and volume fraction, cleanliness, and other factors. In many cases, the thermal conductivity tensor is nearly independent of temperature, and for an anisotropic rectangular heat spreader can be expressed as:

$$\overline{\boldsymbol{k}} = \begin{bmatrix} k_{xx} & k_{xy} & k_{xz} \\ k_{yx} & k_{yy} & k_{yz} \\ k_{zx} & k_{zy} & k_{zz} \end{bmatrix}$$

where the diagonal terms (k_{xx}, k_{yy}, and k_{zz}) are the thermal conductivities in the x-, y-, and z-directions, respectively. The off-diagonal terms (k_{xy}, k_{xz}, k_{yz}, etc.) denote thermal conductivities that couple heat fluxes in one direction with temperature gradients in orthogonal directions. These off-diagonal terms are very small so that they can be neglected in most cases. For this reason, an anisotropic heat spreader can be idealized as an orthotropic heat spreader. For an isotropic material, the thermal conductivity tensor reduces to a single scalar quantity (i.e., $\overline{\boldsymbol{k}} = k_{xx} = k_{yy} = k_{zz} = k$ with zero off-diagonal terms). In contrast, a unidirectional composite containing a square array of continuous fibers (Fig. 1) will display transversely isotropic (TI) bulk thermal conductivity behavior. This type of composite heat spreader exhibits in-plane, isotropic thermal conductivity, i.e., $k_{xx} = k_{yy} = k$. The through-thickness thermal conductivity, k_{zz}, will be a strong function of the fibers' thermal conductivity and volume fraction. Depending on operating conditions and constraints, directionally-dependent thermal transport properties can be advantageous or detrimental for aerospace structural design, especially in the presence of high thermal gradients/loads.

A special class of TI multi-phase heat spreaders consists of uniformly distributed encapsulated structures/fibers or fluid channels oriented perpendicular to the plane of the heat spreader. Such aligned structures/channels can be viewed as 'thermal vias' installed to promote or impede heat transfer and perhaps to provide mechanical stability. Unidirectional composites and heat pipes, when functioning as heat spreaders, can have such thermal vias, resulting in the structure possessing a TI thermal conductivity tensor with conduction bias in the fiber (spreader thickness) direction. Hence, the heat transfer along the 'in-plane' and 'through-thickness' directions are each tailorable. Here, the thermal vias are aligned parallel to the through-thickness direction.



* For transversely isotropic composite, $k_{xx} = k_{yy}$

Fig. 1. A fiber-reinforced, rectangular composite with anisotropic bulk thermal conductivity and uniformly-distributed, unidirectional reinforcements.

The present study provides unique analytical solutions for the steady-state temperature distribution in TI heat spreaders with conduction bias in the thickness direction; such a heat spreader is representative of unidirectional composites or heat pipes with thermal vias (fibers, channels) aligned in the thickness direction. Unlike previous analytical solutions, both the dimensionless, maximum temperature distribution and the thermal spreading resistance are developed as functions of various geometric (e.g., source-to-spreader area ratio, heat spreader thickness) and thermal parameters (e.g., out-of-plane to in-plane thermal conductivity ratio, Biot number). The analyzed heat spreader consists of rectangular geometry and a centrally-located, square heat source located opposite of a plane undergoing uniform free convection. The presented temperature solution is found via a Fourier cosine series expansion [11] and by defining/using an out-of-plane to in-plane thermal conductivity ratio. The accuracy of the analytical temperature solution is benchmarked using steady-state solutions predicted via the ABAQUS finite element (FE) software. Such a comparison is crucial since the FE model may be readily adapted to highly specialized cases involving non-uniform incident heat fluxes, functional gradations in heat spreader material morphologies, and highly tailored heat spreader geometries. The presented solution is intended to serve as a platform for optimizing composite materials or two-phase heat spreaders for various geometric and matrix/constituent combinations.

2. Literature review

Kennedy [5] developed several analytical solutions for the steady-state temperature distribution in a single-phase cylindrical heat spreader with a centrally-applied cylindrical heat source while assuming adiabatic/isothermal boundary conditions. Using a control volume finite difference method, Nelson and Sayers [6] presented two- and three-dimensional (2D and 3D) thermal spreading resistance models for a rectangular heat spreader in Cartesian coordinates and a circular heat spreader in cylindrical coordinates. The two-dimensional rectangular solution agreed well with the 3D rectangular solution for relatively thin heat spreaders. An axisymmetric solution in cylindrical coordinates was found to be insensitive to the heat spreader thickness and accurate (to within 10%) for all Biot numbers investigated. Lee et al. [7] presented an analytical model for the thermal spreading resistance in a rectangular heat spreader (with an axisymmetric cylindrical heat source) subjected to various thermal boundary conditions. The model was flexible enough to reliably predict the response for mixed boundary conditions ranging from isothermal to uniform heat-flux boundary conditions. The solutions agreed well with numerical solutions presented by Nelson and Sayers [6]. Using a Fourier series expansion and Green's functions, Ellison [9] derived an analytical solution for the 3D Poisson's equation governing the temperature distribution of a rectangular heat spreader with various source-to-spreader aspect ratios and Biot numbers. For the special case of a square heat source, the dimensionless thermal spreading resistance was shown to agree reasonably well with results from Lee et al. [7] and Nelson and Sayers [6]. As first proposed by Feng and Xu [10], and later employed by Thompson and Ma [11], the analytical solution for the steady-state temperature distribution in an isotropic, rectangular heat spreader with a centrally-applied, rectangular heat source was derived by solving a modified form of Laplace's equation via a Fourier cosine series. Thompson and Ma [11] incorporated a source-to-spreader area ratio (ratio of the cross-sectional area of a heat