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Estimation of time-dependent wall heat flux from single thermocouple data

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ABSTRACT

Surface mounted sensors disturb the phenomena they are expected to record and are prone to damage in harsh environments, leading to incorrect measurements. The present study examines a robust alternative via the inverse estimation of time-dependent wall heat flux using single thermocouple data, as against an array of embedded thermocouples. The Levenberg-Marquardt algorithm for parameter estimation is employed with an even extension of truncated Fourier series as the trial function for inversion. The inversion error decreases with the length of the signal, number of harmonics in the even extension of truncated Fourier series, and proximity of the thermocouple relative to the boundary of interest. The method is numerically validated as well as experimentally benchmarked, against constant, periodic, and transient wall heat flux data in one-dimensional heat conduction problems and is seen to match the exact solution quite well. The developed method is very well suited for sensor development, particularly for application involving high heat flux.

1. Introduction

The knowledge of boundary heat flux passing through a given domain of interest is required in a variety of engineering applications. Examples can be found in the energy management of buildings, thermal engines, combustors, furnaces, vapor condensation systems, to name a few. Conventional methods for measurement of wall heat flux, primarily making use of a packaged sensor usually demand mounting of the sensor at a particular location/area, where measurement is desired [1–3]. In several situations such as external flows or surface condensation, the mounting of these heat flux sensors on the surface disturbs/disrupts the very thermo-physical phenomena responsible for the ensuing heat flux transport [1]. This signal distortion, possible attenuation, as well as inherent systemic delay in measurement, requires additional calibration of sensors for the estimation of local Wall Heat Flux (WHF). Moreover, in many applications, conditions are so hazardous at the surface of interest, or in the vicinity of it, that the mounted sensor tends to get destroyed over a period of time; for example, in case of fires, hot flame environment, smoldering conditions, hazardous affluent/flue gases inside enclosures, buildings, and furnaces. Hence, there is a need for developing a robust and efficient measurement system for determination of WHF, under normal as well as adverse conditions using embedded thermocouple(s).

Inverse Heat Conduction (IHC) method can be an effective tool for estimation of wall heat flux via measurement of temperature at interior locations [4–7]. It utilizes the knowledge of the 'effect' (usually temperature) to estimate its 'cause' (quantities of interest). One of the main characteristic of an IHC problem is that its solution is not unique, and also not stable to small changes in input data. This ill-posedness of IHC problems makes their solution difficult to obtain; necessitating development and implementation of special solution techniques. Some of the traditional techniques for reformulating the ill-posed IHC problem as an approximate well-posed problem include Tikhonov regularization [4], function estimation approach by Beck et al. [5], iterative regularization by Alifanov [6] or parameter estimation approach [7,8]. These methods have been successfully applied for evaluation of thermophysical properties [9,10], Boundary Condition (BC) [11–13] and strength of heat sources [14]. Recently, stochastic methods have also been frequently used to solve the ill-posed IHC problems. In these methods, data is generated by solving many cases of the direct problem and this data is thereafter used to get optimized estimates.

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Nomenclature

C	trial function (-)
e	% normalized root mean square error (-)
f	sampling frequency (Hz)
I	total number of measurements (-)
J	sensitivity matrix (-)
k	thermal conductivity (W/m-K)
L	length (m)
M	total number of unknown parameters (-)
n	total number of estimated values (-)
N	number of harmonics (-)
P	parameters to be estimated (-)
r	radius (m)
S	sum of squared errors (-)
t	time (s)
T	temperature (K)
x, y	variables (-)
Y	experimental or simulated temperatures (K)

Greek symbols

α	thermal diffusivity (m^2/s)
β	eigenvalues (-)
γ	time period (s)
ε	perturbation (-)
μ	damping parameter (-)
ρ	density (kg/m^3)
τ	signal length (s)
ω	angular frequency (Hz)
Ω	diagonal matrix (-)

Subscripts and superscripts

b	bulk
i, j, m	summation indices
k	iteration index
p	penetration
tc	thermocouple

Some of the commonly used stochastic methods are Genetic algorithms [15,16], Artificial Neural Networks [17,18], Fuzzy algorithms and Bayesian Inferences [19,20]. Other solution methodologies for IHC problems include modal approach for the decomposition of the thermal field by Battaglia [21], input estimation approach based on Karman filtering technique by Tuan et al. [22,23], grey prediction technique by Chiang et al. [24], Laplace transform technique by Monde et al. [25], noninteger identified model by Lohle et al. [26], etc. Comparison of various IHC methods for estimation of WHF from temperature measurements inside a body have been reported in Refs. [27–29].

The Levenberg-Marquardt (L-M) method for parameter estimation has been chosen in the present work due to its applicability for both, linear and nonlinear inverse problems, and robust convergence properties [8]. It was first developed for minimizing the least square norm by Levenberg [30]. Later, the same technique was derived by Marquardt [31] using a different approach. The so called Levenberg-Marquardt technique has been applied to a wide variety of inverse problems for estimation of boundary conditions [32–34], thermophysical properties [35,36] and system heat sources [37]. The technique utilizes an assumed unknown BC, having a combination of unknown parameters and trial functions for solution of the direct problem. Sine, cosine, splines, polynomials, etc. are some functions that can be used as the trial functions. A prior knowledge of the behavior of the unknown BC is quite useful in selection of the trial functions. However, this requirement can be eliminated by using truncated Fourier series as a trial function, for cases where no such prior knowledge is available [38,39]. In spite of this, its use as a trial function and effect of number of harmonics, signal length etc. has not been studied in the literature to the extent that it can be used in practical engineering applications.

In the present paper, we propose a generalized IHC method based on L-M algorithm, with an even extension of truncated Fourier series as the trial function, to estimate WHF of any form, using recorded temperature data from a single location. Section 2 presents definition and solution methodology of the developed IHC method for 1-D unsteady heat conduction problem. Sensitivity analysis has been performed in Section 3 to identify most suitable thermocouple location and delineate the effect of heat conduction medium on inverse estimation. Section 4 provides validation for the developed IHC method using simulated temperature data and Section 5 describes its potential for use in practical applications. Finally, results of the inverse estimation are presented in Section 6, followed by discussion in Section 7, where the effect of thermocouple location, signal length, sampling frequency and number of harmonics, on the estimation of WHF are reported.

2. Mathematical modeling

2.1. Problem formulation

A 1-D unsteady heat conduction is considered in a stainless steel test plate, as shown in Fig. 1(a). The governing differential equation, BCs and initial condition applied to the test plate are:

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \quad \text{in } 0 < x < L,\ t > 0 \tag{1}$$

$$q'' = -k\frac{\partial T}{\partial x} = f(t) \quad \text{at } x = 0,\ t > 0 \tag{2}$$

$$T = T_0 \quad \text{at } x = L,\ t > 0 \tag{3}$$

$$T = T_0 \quad \text{at } t = 0,\ \forall x \tag{4}$$

Here, $f(t)$ is an unknown WHF specified at $x = 0$ and can be of any general form. T_0 is a constant temperature at $x = L$, which is also the uniform initial temperature inside the test plate. The BCs applied to the test plate, along with its relevant dimensions are also shown in Fig. 2.

Even though we are presenting here the IHC method using 1-D unsteady heat conduction problem with prescribed BCs at both ends, the methodology described in this paper is generic in nature. Its forward model can be modified to solve various heat transfer problems, such as semi-infinite, multidimensional and conjugate types. Solving the 1-D problem via semi-infinite approach will eliminate the requirement for prior knowledge of rear BC ($x = L$). However, most of the heat transfer geometries encountered in practical applications have finite dimensions and cannot be approximated as semi-infinite, especially for long duration studies. Therefore, we have presented here the validation and application of IHC method pertinent to practical engineering domains.