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A lattice Boltzmann algorithm for simulating conjugate heat transfer through virtual heat capacity correction

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ABSTRACT

A fundamental problem for lattice Boltzmann method (LBM) to simulate conjugate heat transfer between solid and fluid is that lattice Boltzmann equation (LBE) can only retrieve standard energy equation when solid has the same heat capacity (ρc) with that of fluid. When their heat capacities are different, the continuity of heat flux cannot be kept at the interface for conventional LBM. In present work, a lattice Boltzmann algorithm for simulating conjugate heat transfer through virtual heat capacity correction method is proposed to solve this problem. This algorithm, at first, assume that the solid has the same heat capacity (ρc) with that of fluid to keep the continuity of heat flux at the interface between solid and fluid, after that modify the temperature field and the distribution function according to real data of heat capacity of fluids or solids at the end of each time step. This algorithm also fits to conjugate heat conduction problems between solid and solid as long as we take one of the solids as a kind of fluid that does not flow. Several test cases including both steady and transient conjugate heat transfer with flat or curved interface are calculated to validate the present method. The results show that the proposed method has a very good performance in simulating solid-fluid and solid-solid conjugate heat transfer, which is easy in implementation, capable for solving both steady and transient heat transfer problems. Also, the present method can deal with curved interface easily without a necessary to find the normal direction of interface. However, this method suffers a stability problem while the heat capacity (ρc) of fluid is larger than that of solid, which will be eliminated in future work.

1. Introduction

Heat transfer across the interface between solid and fluid or solid and solid is one of fundamental issues in multiphase heat transfer. It has numerous applications in engineering problems such as cooling of electronic devices [1,2], lab-on-a-chip [3], micro-TAS [4], micro fuel cells [5], and microchannels [6] etc.

Depending on the numerical methods for solving the governing N-S and energy equations, various approaches have been proposed to keep temperature and energy (e.g. heat) flux continuity at the interface between the two domains of different solids, solid and fluid, or different fluids [7,8]. In the category of conventional numerical methods, Finite Volume Method (FVM), Finite Difference Method (FDM), Finite Element Method (FEM) have been successfully applied to simulate conjugate heat transfer problems. For example, Fiebig et al. [9] made a numerical investigation on conjugate heat transfer in a finned-tube element with FVM. Ha et al. [10] simulated three-dimensional natural convection and conduction in a differentially heated cubic enclosure with a heat-generating cubic conducting body with FVM. By taking FDM, Korichi et al. [11] made a numerical simulation on conjugate heat transfer in a rectangular channel with mounted obstacles on upper and lower walls. More systematic introduction can be found in the review given by Dorfman and Renner [12] and the references therein.

Based on the particle distribution function of discrete velocities, lattice Boltzmann method (LBM) has been developed rapidly in recent years. As proved by some previous researchers, LBM is an effective and very simple method in simulating the solid-fluid flow systems [13–17]. Up to date, there are some researchers tried to spread the use of LBM to solid-fluid conjugate heat transfer problems. In the category of LBM, some improved SIMPLE-like algorithms were adopted to simulate conjugate heat transfer problem [18–20]. These methods assume that the solid has the same heat capacity ρc (ρ and c are,

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respectively, the density and the specific heat capacity) with that of the fluid, therefore, can only recover the steady energy equation. Also several difference methods have been proposed [21–23] to deal with conjugate heat transfer problems, in which both an equal temperature and a continuous heat flux at the interface of solid and fluid can be well guaranteed. However, the raised examples in those methods are only for solids with a square shape at steady state. Recently, Li et al. [24,25] proposed a new difference method to ensure temperature and energy flux are continuous at a curved interface, but the difference formula is so complex that not easy to implement, especially for the cases when the normal direction of interface is not easy to find. Karani and Huber [26] suggested using an additional energy source term which introduces the jump conditions at the interface between two different phases to recover the energy equation approximately with first-order accuracy, this method can also be used to simulate conjugate heat transfer with curved interface approximately. This idea was developed by Rihab et al. [27], in their paper, enthalpy rather than temperature is solved directly to make the expression of additional source term is easy to calculate. However, their method is only applicable for conjugate heat conduction. Another promising approach was proposed by Mohamad et al. [28], in their method, the continuity of heat flux at the interface is ensured by coupling temperature difference with a new scaling law of energy distribution function. It is suitable for steady and unsteady conjugate heat transfer between two different immiscible media, what confines this method is that it requires the interface must locate at computational nodes.

Considering the complex geometry, and the difference of thermal properties between the two sides of an interface, the numerical simulations for conjugate heat transfer are generally not easy to be implemented with LBM. A more accurate, reliable, and simpler lattice Boltzmann algorithm is still desired. In this paper, a novel lattice Boltzmann algorithm called by virtual heat capacity correction method for simulating solid-fluid or solid-solid conjugate heat transfer is introduced, which is very easy to be implemented, and capable for solving both steady and transient heat transfer problems. Also, the present method can deal with curved interface easily, which does not need to find the normal direction of an interface. The remaining part of this paper is organized as follows. In Section 2, the conventional LBM is briefly reviewed and the virtual heat capacity correction method is introduced in detail. To validate the present method, seven test cases are given in Section 3. The one-dimensional heat conduction problem is firstly considered in Section 3.1; followed by two unsteady heat conduction problems in a three-layer stratified medium in Section 3.2; thirdly, the heat conduction in a two-layered annulus is calculated in Section 3.3; the next is unsteady heat conduction in a square medium containing discrete circular media in Section 3.4; and finally, two natural convection problems in a square cavity with uniformly distributed solid blocks are simulated in Section 3.5 and Section 3.6 separately. In Section 4, summary and conclusions are addressed.

2. Numerical method

In this paper, the single-relaxation-time (BGK) D2Q9 lattice Boltzmann model is used, the two lattice Boltzmann equations for simulating fluid flow and heat transfer, respectively, can be written as follows [29]:

$$f_i(\vec{x}+\vec{e}_i\Delta t, t+\Delta t) - f_i(\vec{x},t) = -\frac{1}{\tau_f}\left[f_i(\vec{x},t) - f_i^{(eq)}(\vec{x},t)\right] + F_i(\vec{x},t)\Delta t \tag{1}$$

$$g_i(\vec{x}+\vec{e}_i\Delta t, t+\Delta t) - g_i(\vec{x},t) = -\frac{1}{\tau_g}\left[g_i(\vec{x},t) - g_i^{(eq)}(\vec{x},t)\right] \tag{2}$$

where f_i and g_i are the distribution functions of density and energy respectively, $f_i^{(eq)}$ and $g_i^{(eq)}$ are their corresponding distribution functions at equilibrium state, respectively. F_i is the discrete force term, τ_f and τ_g are the relaxation times related to, respectively, kinematic viscosity and thermal diffusivity, t is the time in simulation. The corresponding equilibrium distribution functions of $f_i^{(eq)}$ and $g_i^{(eq)}$ are expressed as follows.

$$f_i^{(eq)} = w_i\rho\left[1 + 3(\vec{e}_i\cdot\vec{u}) + \frac{9}{2}(\vec{e}_i\cdot\vec{u})^2 - \frac{3}{2}\vec{u}\cdot\vec{u}\right] \tag{3}$$

$$g_i^{(eq)} = w_i T\left[1 + 3(\vec{e}_i\cdot\vec{u}) + \frac{9}{2}(\vec{e}_i\cdot\vec{u})^2 - \frac{3}{2}\vec{u}\cdot\vec{u}\right] \tag{4}$$

And the discrete forcing terms of $F_i^{(eq)}$ is:

$$F_i(\vec{x},t) = \omega_i\left(1-\frac{1}{2\tau_f}\right)\left[\frac{\vec{e}_i - \vec{u}(\vec{x},t)}{c_s^2} + \frac{\vec{e}_i\cdot\vec{u}(\vec{x},t)}{c_s^2}\vec{e}_i\right]\cdot\vec{F}(\vec{x},t) \tag{5}$$

where $c_s = 1/\sqrt{3}$ is the lattice sound speed. The bulk properties of density, velocity and temperature are obtained by:

$$\rho = \sum_i f_i \tag{6}$$

$$\rho\vec{u} = \sum_i f_i\vec{e}_i + \frac{\vec{F}}{2}\Delta t \tag{7}$$

$$T = \sum_i g_i \tag{8}$$

The relaxation times τ_f and τ_g related to kinematic viscosity and thermal diffusivity of fluid, respectively, can be given by the following equations:

$$\upsilon = \left(\tau_f - 0.5\right)c_s^2\Delta t \tag{9}$$

$$\alpha = (\tau_g - 0.5)c_s^2\Delta t \tag{10}$$

By using Chapman-Enskog expansion [30], this lattice Boltzmann model can recover the following energy equation:

$$\frac{\partial T}{\partial t} = \nabla\cdot(\alpha\nabla T) - \nabla\cdot(\vec{u}T) \tag{11}$$

where α is the thermal diffusivity, $\vec{u}$ is the velocity. For solid, there is no convection term $\nabla\cdot(\vec{u}T)$ and the model can recover the energy equation of transient heat conduction as follows:

$$\frac{\partial T}{\partial t} = \nabla\cdot(\alpha\nabla T) \tag{12a}$$

The above equation can also be written as:

$$\rho c\frac{\partial T}{\partial t} = \nabla\cdot(k\nabla T) + \rho c\cdot\nabla\left(\frac{1}{\rho c}\right)\cdot(k\nabla T) \tag{12b}$$

When ρc do not change with position, the energy equation for