International Journal of Thermal Sciences 116 (2017) 45–51

Contents lists available at ScienceDirect

International Journal of Thermal Sciences

journal homepage: www.elsevier.com/locate/ijts

Exact closed form solutions to nonlinear model of heat transfer in a straight fin

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ARTICLE INFO

Article history:
Received 26 September 2016
Received in revised form
19 January 2017
Accepted 20 January 2017
Available online 21 February 2017

Keywords:
Exact analytical solutions
Unique solution
Dual solutions
Nonlinear fin equation
Heat transfer
Thermal conductivity

ABSTRACT

This article is concerned with the exact closed form solutions for heat transfer in a straight fin when the thermal conductivity and heat transfer coefficients are temperature dependent so that conduction and heat transfer terms have the form of strong nonlinearity given by power laws. It is assumed the shape for a straight fin with a rectangular profile is linear dependent with respect to fin thickness. A full discussion and exact analytical solution in the implicit form is given for further physical interpretation and it is proved that three possible cases may occur: there is no solution to the problem, the solution is unique and the solutions are dual depending on the values of the parameters of the model.

1. Introduction

Heat transfer problems occurring in extensive applications in mechanical engineering, theoretical physics and other fields in engineering are generally reduced to nonlinear differential equations through a process of mathematical modeling. The heat transfer through the straight fins constitutes such a real nonlinear problem being a recent active research subject in the literature. The efficiency of heating systems is generally succeeded by fins, particularly, in the applications of air conditioning, air-cooled craft engines, refrigeration, cooling of computer processor, cooling of oil carrying pipe line. Numerous contributions, therefore, have been made in the heat transfer analysis of fins. Attention has been paid to models describing longitudinal trapezoidal fins [1], rotating radial fins of rectangular and other profiles [2], radial fins of rectangular profile [3], rectangular fins [4], radial rectangular fins [5] and longitudinal fins of rectangular profile [6]. In this paper, we revisit heat transfer in straight fins with nonlinear type of temperature-dependent thermal conductivity and heat transfer coefficient and then discuss about the multiplicity of the solutions, see Fig. 1.

2. The fin problem formulation

A straight one-dimensional fin having a constant cross-sectional area A_c is considered. We denote the perimeter and the length of fin by P and L respectively. The fin is attached to a base surface of temperature T_b and extends into a fluid of temperature T_a. Furthermore, suppose that the small amount of heat transfer through the tip end is negligible. The one-dimensional steady state heat balance equation in dimensional form may be written as [4,6–8].

$$A_c \frac{\mathrm{d}}{\mathrm{d}X}\left(f(X)K(T)\frac{\mathrm{d}T}{\mathrm{d}X}\right) - Ph(T - T_a) = 0, \quad 0 < X < L, \tag{1}$$

where h is the non-uniform heat transfer coefficient depending on the temperature. The heat transfer coefficient might depend on the temperature and usually can be expressed as a power-law form

$$h = h(T) = h_b\left(\frac{T - T_a}{T_b - T_a}\right)^n, \tag{2}$$

where h_b is the heat transfer coefficient at the base temperature. The exponent n depends on the heat transfer mode. The value of n can vary in a wide range between −4 and 5 [9,10]. For example, the exponent n may take the values −4, −0.25, 0, 2, and 3, indicating

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http://dx.doi.org/10.1016/j.ijthermalsci.2017.01.028
1290-0729/

Nomenclature

T	temperature
T_b	base temperature
T_a	fluid temperature
h	non-uniform heat transfer coefficient
P	perimeter of fin
L	fin length
m	fitting coefficient in the power-law heat transfer coefficient
k	thermal conductivity
A_c	cross-sectional area
h_b	heat transfer coefficient at the base temperature
X	dimensional space coordinate
M	dimensionless fin parameter
n	exponent in Eq. (2)
x	non-dimensional space coordinate
u	auxiliary changing variable
C	integral constant

Greeks symbols

ζ	fin tip temperature
η	fin efficiency
ϕ	heat transfer rate
θ	non-dimensional temperature
θ_0	$\theta'(0)$ defined in Eq. (16)

the fin subject to transition boiling, laminar film boiling or condensation, convection, nucleate boiling, and radiation into free space at zero absolute temperature, respectively. In many engineering applications, the thermal conductivity K in Eq. (1) might be assumed as a power law function of temperature [4,7].

$$K = K(T) = K_a\left[\lambda(T-T_a)^m\right], \tag{3}$$

and the shape for a straight fin with a rectangular profile is linear dependent with respect to fin thickness, i.e. [4,7].

$$f(X) = \frac{\delta}{2}. \tag{4}$$

In Eq. (3), K_a is the thermal conductivity of the fin at the ambient temperature and λ is the parameter describing the dependence on the temperature. Now, the relevant dimensionless energy equation gives [6,8].

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{\delta}{2}k(\theta)\frac{\mathrm{d}\theta}{\mathrm{d}x}\right] - N^2\theta^{n+1} = 0, \quad 0<x<1, \tag{5}$$

where

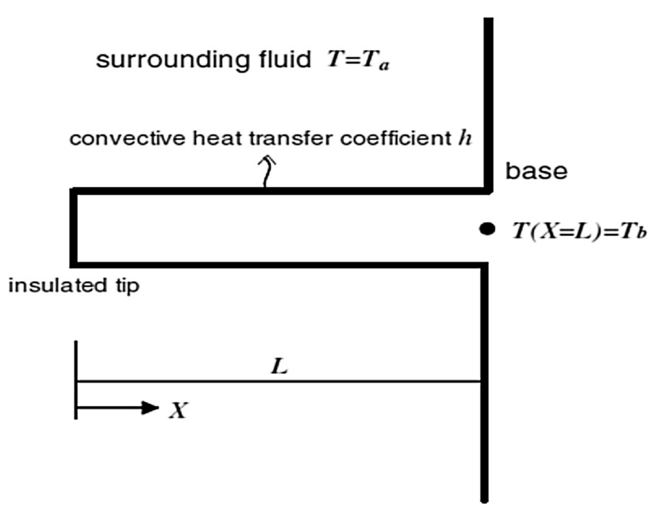

Fig. 1. Description of a one-dimensional fin.

$$\theta = \frac{T-T_a}{T_b-T_a}, \quad x=\frac{X}{L}, \quad k=\frac{K}{K_a}, \quad \beta=\lambda(T_b-T_a)^m, \quad N^2 = \frac{h_bPL^2}{K_aA_c}, \quad k(\theta)=\beta\theta^m. \tag{6}$$

The dimensionless boundary conditions are

$$\theta'(0) = 0, \quad \text{at the tip}, \tag{7}$$

$$\theta(1) = 1, \quad \text{at the base}. \tag{8}$$

Denoting $M^2 = \frac{2N^2}{\delta\beta}$, the governing dimensionless energy equation and boundary conditions (5)–(8) are then given simply by

$$\theta^m\theta'' + m\theta^{m-1}\left(\theta'\right)^2 - M^2\theta^{n+1} = 0, \tag{9}$$

$$\theta'(0) = 0, \quad \theta(1) = 1. \tag{10}$$

As a brief review of the problem (9)–(10) when $m = 0$ (i.e. the thermal conductivity is constant) in the literature, we can recall [11,12] where authors have applied homotopy perturbation method [13,14] and variational iteration method. M.S.H. Chowdhury and I. Hashim [15] have used the homotopy perturbation method so that they could give approximate solutions for positive exponent of θ. Chang [16] has applied Adomian decomposition method [17,18]. Abbasbandy [19] has demanded homotopy analysis method (HAM) (the interest readers are referred to [20–34] and Refs. therein for more information about HAM). Moreover, the exact analytical solution has been obtained by Abbasbandy and Shivanian in Ref. [35].

The exact analytical form of the efficiency and the effectiveness of fin as well as the rate of heat transfer, for the problem (9)–(10), has been given in Ref. [36]. In addition, a little different type of the problem (9)–(10) has been solved approximately using Taylor series expansion method by Kim and Huang [8]. The homotopy analysis method (HAM) has been adopted to evaluate the analytical approximate solutions of the same nonlinear fin problem in Ref. [6]. Also, the authors of [37] presented approximate solutions of this kind of problem obtained by the differential transform method (DTM [38–40]). In Ref. [4], the authors used Lie symmetry techniques to exclude some exact solutions for some special cases of the problem (9)–(10). Turkyilmazoglu presented the exact solutions of the problem (9)–(10) for some special cases for m using erudite change of variable and modified Bessel functions.

In this paper, it is shown the governing strongly nonlinear differential equations (9) and (10) for entire range of $n \in \mathbb{R}$ with $m \geq 0$,