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Exact temperature field in a slab with time varying ambient temperature and time-dependent heat transfer coefficient

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ABSTRACT

An analytical solution is presented for one-dimensional heat conduction of a slab with time-varying ambient temperature and time-dependent heat transfer coefficient at the same boundary for the first time. The solution is obtained by using the shifting function method. After a shifting function is specified and series expansion is performed for the boundary value problem, the solution is generated. When limiting studies on either constant ambient temperature or constant heat transfer coefficient are conducted, the present solutions are proven to be identical to those in the literature. Through our investigation on numerical examples, this study shows that the three-term approximation solution can get an error within 1%. Two examples, both exponential and periodical heating or cooling on the slab, are utilized to demonstrate the influence of physical parameters regarding different convective heat transfer on temperature profiles. The difference in temperature field between the slab under time-varying temperature environment and the slab under constant ambient temperature is obvious.

1. Introduction

Transient heat conduction plays a central role in various engineering applications, such as heating and cooling of metal billets, heat treatment of metals by quenching, cooling of internal combustion engines, starting and stopping of many heat exchange units in power installation, etc. Because of the complexity of transient heat conduction, the one-dimensional heat transfer of media with time-dependent heat transfer coefficient (HTC) is often taken as the research subject. Two kinds of problems are investigated. One is a direct problem that both boundary conditions and initial condition are prescribed, and the other one is an inverse problem that temperature at interior point of media is known. Generally speaking, the direct problem occurs mainly in design practice while the inverse problem occurs in the analysis of experimental data. The analytical solutions obtained from direct problems, however, can be utilized to strengthen the development of solution techniques for inverse problem.

When direct problems of one-dimensional heat conduction of a slab are studied, the slab is usually considered with time-dependent HTC at one end together with insulated boundary at the other end. The problem is attributed to a boundary value problem, which could be solved by some numerical and/or approximated methods [1–4]. In 2010, an analytical solution was first developed via the shifting function method by Chen et al. [5]. Using a dimensionless temperature as temperature subtracting an ambient temperature over the ambient temperature, they proposed a series solution without using the method of variation of parameter. Later, in a companion study, Tu and Lee [6] took a different shifting function, which has the physical meaning to obtain another series solution. Recently, Lee and Tu [7] developed a closed solution for the same heat transfer problem but the slab with non-homogenous time-dependent boundary condition at one end and homogenous boundary condition with time-dependent HTC at the other end. In the mean time, the authors [8] presented a closed solution for the heat transfer in a hollow cylinder with time-dependent boundary condition and time-dependent HTC at different surfaces.

Furthermore, when inverse heat conduction problems [9,10] on different media are considered, the interior temperature is measured by experiment and the time-dependent HTC is unknown. Several inverse schemes [11–16] for determining the time-dependent HTC have proposed. Chantasiriwan [11] used the sequential function specification method to determine the time-

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dependent HTC of one-dimensional heat conduction of a slab. Zueco et al. [12] applied the same method but combining with network simulation method in the inverse estimation of the time-dependent HTC with internal heat generation. Onyango et al. [13] determined the time-dependent HTC in one-dimensional transient heat conduction from a non-standard boundary measurement. When two-dimensional transient inverse heat conduction problem is studied, Chen and Wu [14] utilized a hybrid inverse scheme of the Laplace transform, finite difference and least-square methods to predict the HTC distribution on a boundary surface. Also, when the studied medium is a hollow cylinder, Su and Hewitt [15] used Alifanov's iterative regularization method to estimate the time-varying HTC of forced-convective flow boiling over the outer surface of a heater tube. Later, Moitsheki [16] performed for an unsteady nonlinear heat diffusion problem modeling thermal energy storage in a slab, a hollow cylinder and a sphere, with a temperature-dependent power law thermal conductivity and time-dependent HTC.

According to aforementioned literature [1–16], because of the limitation on solution methodology, the investigated systems for either direct or inverse problems are all confined on constant ambient temperature at the convective surface. It is well known that the convective HTC is a property affected by the characteristic of flow field. Therefore, the assumption of constant ambient temperature is unreasonable in real world. To our knowledge, to date, no studies have attempted to analytically solve the heat conduction for slabs with time-varying ambient temperature and time-dependent HTC at the same boundary surface.

This paper utilizes the shifting function method [5–8,17–19] to develop an analytical solution for heat transfer of a slab with zero heat flux at one end and with time-dependent HTC and time-varying ambient temperature at the other end for the first time. No inverse method is implemented in the work. The analytical solution can be reduced to obtain both the solution of boundary condition with constant ambient temperature and the solution of boundary condition with constant HTC. Examples with constant initial temperatures reveal the solution procedure and numerical results for limiting studies are compared with those in the literature. Finally, the influence of different kinds of time-varying ambient temperature on the temperature distribution in the slab is also studied.

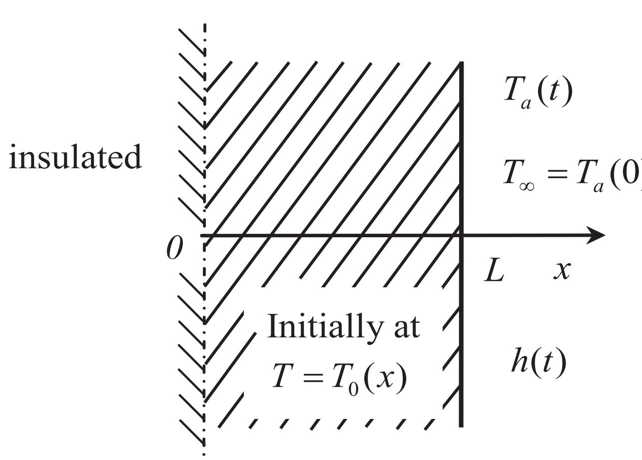

Fig. 1. One-dimensional heat transfer system of a slab with time-varying ambient temperature and time-dependent heat transfer coefficient at one end.

2. Mathematical modeling

Consider the one-dimensional heat conduction in a slab with boundary conditions of different kinds at two ends, as shown in Fig. 1. The slab is insulated at one end whereas the other end is subjected to a time-varying ambient temperature and time-dependent HTC simultaneously. No heat generated within the slab is assumed. The governing differential equation of the system is

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}, 0 < x < L, \quad t > 0, \tag{1}$$

where, T is the temperature, x is the spatial-domain variable, α is the thermal diffusivity, t is the time, L is the thickness of the slab. The boundary and initial conditions now are

$$\frac{\partial T}{\partial x} = 0, \text{ at } x = 0, \tag{2}$$

$$-k\frac{\partial T}{\partial x} = h(t)[T - T_a(t)], \text{ at } x = L, \tag{3}$$

$$T(x,\ 0) = T_0(x), \text{ at } t = 0. \tag{4}$$

Here k is the thermal conductivity, $h(t)$ is the time-dependent HTC, $T_a(t)$ is the ambient temperature function which varies with time, and $T_0(x)$ is the initial temperature function of the slab.

First, we define the following dimensionless quantities:

$$\theta = \frac{T - T_r}{T_\infty - T_r},\ X = \frac{x}{L},\ \tau = \frac{\alpha t}{L^2},\ Bi(\tau) = \frac{h(t)L}{k},$$
$$\theta_\infty(\tau) = \frac{T_a(\ t) - T_r}{T_\infty - T_r},\ \psi(\tau) = Bi(\tau)\theta_\infty(\tau),\ \theta_0(X) = \frac{T_0(x) - T_r}{T_\infty - T_r}, \tag{5}$$

where T_r is a constant reference temperature, T_∞ is an initial fluid temperature which is equal to $T_a(0)$, τ and $Bi(\tau)$ denote the Fourier number and the Biot function, respectively. With these dimensionless quantities, the associated governing differential equation along with boundary and initial conditions now become

$$\frac{\partial^2 \theta}{\partial X^2} = \frac{\partial \theta}{\partial \tau}, \text{ in } 0 < X < 1, \tau > 0, \tag{6}$$

$$\frac{\partial \theta}{\partial X} = 0, \text{ at } X = 0, \tag{7}$$

$$\frac{\partial \theta}{\partial X} + Bi(\tau)\theta = \psi(\tau), \text{ at } X = 1, \tag{8}$$

$$\theta = \theta_0(X), \text{ at } \tau = 0. \tag{9}$$

Next, to remain the boundary condition of the third kind at $X = 1$ in the following analysis, the Biot function is expressed as

$$Bi(\tau) = \delta + F(\tau), \tag{10}$$

where constant δ is the initial value of $Bi(\tau)$. Substitution of the Biot function into equation (8) leads to

$$\frac{\partial \theta}{\partial X} + \delta\theta = -F(\tau)\theta + \psi(\tau), \text{ at } X = 1. \tag{11}$$