International Journal of Thermal Sciences 116 (2017) 310–319

Contents lists available at ScienceDirect

International Journal of Thermal Sciences

journal homepage: www.elsevier.com/locate/ijts

Heat transfer correlations for natural convection in inclined enclosures filled with water around its density-inversion point

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ARTICLE INFO

Article history:
Received 18 April 2016
Received in revised form
10 February 2017
Accepted 8 March 2017

Keywords:
Natural convection of water
Tilted enclosure
Differential heating
Density-inversion point
Numerical analysis
Dimensionless correlations

ABSTRACT

Natural convection in tilted square cavities filled with water, having one side cooled at 0 °C and the opposite side heated at a temperature ranging between 8 °C and 40 °C, is studied numerically for different cavity widths in the hypothesis of temperature-dependent physical properties, exploring the full range of inclination angles. A computational code based on the SIMPLE-C algorithm is used to solve the system of the mass, momentum and energy transfer governing equations. It is found that, as the inclination angle is increased starting from the cooling-from-below configuration, the heat transfer rate keeps substantially constant until the breakdown of the upper fluid stratification occurs. Thereafter, the heat transfer performance increases steeply up to reaching a peak at an optimal tilting angle, which increases with decreasing both the cavity width and the temperature of the heated wall. Furthermore, when the combination of the cavity width and the temperature of the heated wall is such that at small tilting angles the buoyancy force in the water layer confined between the cooled bottom wall and the density-inversion isotherm is that required for the onset of convection, or just higher, the asymptotic solution is periodical. A number of dimensionless correlations are developed for the prediction of both the optimal tilting angle and the heat transfer rate across the enclosure.

1. Introduction

Enclosed natural convection of water near its density-inversion point has been the subject of many experimental and numerical investigations performed over the past decades, due to its relevance to several science and engineering applications, such as ice forming and melting, crystal growth, and cooling energy storage, to name a few. Actually, besides a number of papers related to infinite horizontal layers with either stress-free or rigid top and bottom boundaries differentially heated above and below the density-inversion temperature of approximately 4 °C at atmospheric pressure [1–6], most studies were executed using either horizontal or vertical enclosures [7–33]. A summary of these publications is presented in Table 1, in which indications on the research method, the cavity orientation with respect to the gravity vector, φ (where $\varphi = 0°$ corresponds to the cooling-from-below configuration), the height-to-width aspect ratio of the enclosure, A, and the density-inversion parameter, θ_{inv}, are reported. Recall that the definition of the density-inversion parameter is $\theta_{inv} = (t_{inv} - t_c)/(t_h - t_c)$, where t_{inv} is the density-inversion temperature, while t_h and t_c are the temperatures of the heated and cooled walls. The basic result of all these studies is that the flow structures arising from the density inversion markedly affect the heat transfer rate, which can considerably decrease compared with that typical for a fluid having a monotonic relationship between density and temperature. In particular, when the cavity is differentially heated at sides, the formation of a two-cell flow pattern occurs, whereas, when a horizontal differential heating is imposed, the formation of a stably stratified fluid layer takes place near the top wall. In both cases, direct convection between the hot and cold walls is partially or totally prevented, which limits the amount of heat transferred across the cavity.

The inclined configuration, in contrast, has been much less investigated. The first studies, which date back to 1984, were authored by Inaba and colleagues [34–36], who performed experiments and numerical simulations using a square cavity of width 0.015 m, with one side cooled at $t_c = 0$ °C and the opposite side heated at $t_h = 4$ °C–20 °C ($\theta_{inv} = 0.2–1$), for seven tilting angles in the range $\varphi = 0°–180°$. For $t_h < 8$ °C, the Nusselt number Nu was found to increase gradually with increasing φ up to reaching a peak

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http://dx.doi.org/10.1016/j.ijthermalsci.2017.03.008
1290-0729/

Nomenclature

a_i	i-th polynomial coefficient
c	specific heat at constant pressure, J/(kg K)
$\mathbf{g}$	gravity vector, m/s^2
h	average coefficient of convection, W/(m^2 K)
$\mathbf{I}$	unit tensor
k	thermal conductivity, W/(m K)
Nu	Nusselt number
p	pressure, Pa
Q	heat transfer rate, W
q	heat flux, W/m^2
Ra	Rayleigh number
T	period of oscillation, s
t	temperature, °C
U	x-wise velocity component, m/s
$\mathbf{V}$	velocity vector, m/s
V	y-wise velocity component, m/s
W	width of the enclosure, m
x, y	Cartesian coordinates, m

Greek symbols

ϕ	generic physical property
φ	tilting angle, deg or rad
μ	dynamic viscosity, kg/(m s)
θ_{inv}	density-inversion parameter
ρ	mass density, kg/m^3
τ	time, s
ψ	stream function, kg/(m s)

Subscripts

c	cooled wall, at the temperature of the cooled wall
h	heated wall, at the temperature of the heated wall
i	initial
inv	density-inversion point
opt	optimal value
0	at 0 °C

at about $\varphi = 30°–60°$, beyond which it decreased significantly. For $t_h = 8$ °C, Nu was found to increase weakly with increasing φ up to reaching a peak at about $\varphi = 90°$, beyond which it decreased following a similar trend as that for $\varphi < 90°$. Finally, for $t_h > 8$ °C, Nu was found to increase noticeably with increasing φ until reaching a smooth maximum at about $\varphi = 120°$. Indeed, regarding the data obtained for $t_h = 8$ °C, it must be noticed that the peak of Nu detected numerically at $\varphi = 90°$ is not justified at all, taking into account that the set of governing equations was formulated assuming temperature-dependent physical properties. Possibly, as also stated by Osorio et al. [37] in a technical note revisiting the results published by Inaba and co-workers, the dependence on temperature was assumed only for the density, while all the other physical properties were calculated at the mean temperature between t_c and t_h. Such a simplified approach, which can give rise to non negligible inconsistencies, was followed more recently by Rodionov [38], who executed numerical simulations using a cubic cavity having one side cooled at $t_c = 0$ °C and the opposite side heated at $t_h = 4$ °C–20 °C in the range of tilting angles $\varphi = 0°–180°$, for two Grashof numbers corresponding to side-lengths of 0.0085 m–0.0365 m, whose results were substantially in line with those previously obtained by Inaba and colleagues. In none of these studies heat transfer correlating equations were proposed.

The above review of the existing literature shows that the data available for the inclined configuration, other than showing some inaccuracies, are very limited. For this reason, in the present paper a

Table 1
Summary of the studies performed on natural convection of water around 4 °C in either horizontal or vertical rectangular enclosures.

Year	Author(s)	Method	A	θ_{inv}	φ
1964	Townsend [7]	Exp	0.5	0.16	0°
1971	Desai and Forbes [8]	Num	1–3	0.5	90°
1972	Watson [9]	Num	1	0.4–0.66	90°
1978	Seki et al. [10]	Exp/num	1–20	0.33–1	90°
1979	Robillard and Vasseur [11]	Num	1	0.33–1	90°
1986	Lankford and Bejan [12]	Exp	5	~0.5	90°
1987	Lin and Nansteel [13]	Num	1	0.4–1	90°
1989	Ivey and Hamblin [14]	Exp/num	0.1–0.5	0.5	90°
1992	Braga and Viskanta [15]	Exp/num	0.5	0.2–0.5	90°
1993	Ishikawa et al. [16]	Num	0.66–5	0.33–1	90°
1993	Bennacer et al. [17]	Exp/num	0.66–4	0.33–0.66	90°
1994	McDonough and Faghri [18]	Exp/num	0.75	0.5–0.8	90°
1994	Tong and Koster [19]	Num	0.25–10	0.33–1	90°
1995	Nishimura et al. [20]	Num	1.25	0.4–1	90°
1999	Tong [21]	Num	0.125–100	0.33–1	90°
2000	Ishikawa et al. [22]	Num	1	0.2–0.8	90°
2001	Ho and Tu [23]	Exp/num	8	0.4–0.5	90°
2001	Zubkov and Kalabin [24]	Num	1	0.5	180° (*)
2002	Zubkov et al. [25]	Num	1	0.5	180° (*)
2004	Moraga and Vega [26]	Num	1	0.4	90°
2005	Hossain and Rees [27]	Num	0.5–2	0.5	90°
2008	Sivasankaran and Ho [28]	Num	0.5–2	0.4–1	90°
2012	Li et al. [29]	Num	0.25	0.3	180°
2015	Cianfrini et al. [30]	Num	1	0.5	180°
2015	Li et al. [31]	Num	0.5	0.1–0.6	180°
2015	Corcione and Quintino [32]	Num	1	0.05–0.5	0°
2015	Cianfrini et al. [33]	Num	1	0.13–0.4	0°

(*)results applicable also to the cooling-from-below configuration ($\varphi = 0°$).