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# Onset of Darcy-Brinkman convection with a uniform internal heat source and vertical throughflow

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ABSTRACT

The problem of thermal convection in a horizontal fluid-saturated porous layer is examined, where the flow is governed by the Brinkman extension of Darcy's law. A uniform internal heat source and vertical throughflow are also considered. The linear and nonlinear stability analyses are performed in order to determine the stability characteristics of the system. The linear and nonlinear thresholds give good agreement in the absence of vertical throughflow. However, it is shown that there are potential regions of sub-critical instabilities for increasing values of internal heat source parameter *Q*, Péclet number *Pe* and Darcy number *Da*.



## 1. Introduction

The flow and heat transfer in porous media has been a highly active area of research for the past few decades due to broad range of applications in engineering field as well as in the hydrology of ground water, food processing, geophysics, and petroleum reservoirs etc.

Horton and Rogers [1] and Lapwood [2] were first to study the flow and heat transfer in a porous media. These studies are based on the Darcy law, which neglects the effects of inertial forces and solid boundary. However, in the case of highly porous materials, where these effects are significant. The permeability $K$ in the Brinkman equation is such that the equation reduces to a form of the Navier-stokes equation as $K \to \infty$ and to the Darcy equation as $K \to 0$. Brinkman [3] calculated the viscous force exerted by fluid flow on a dense swarm of particles embedded in a porous medium which lead to the formation of the Darcy-Brinkman model. In Brinkman [3] the simultaneous effects of boundary and inertia are discarded. Tam [4] specified that when the spatial length scale is much higher than $\frac{1}{\alpha}$ (where $\alpha^2 = \frac{\mu}{K\tilde{\mu}}$: $\mu$ is the viscosity, $\tilde{\mu}$ is the effective viscosity, $K$ is the permeability), the term $\mathbf{v}$ (linear in fluid flow $\Delta\mathbf{v}$) is insignificant in comparison with the linear term $\mathbf{v}$. In this case the Darcy-Brinkman model reduces to the Darcy law. Later, Vafai and Tien [5] analyzed the inertia and solid boundary effects on flow and heat transfer in porous media, in which the local volume averaging technique was used to establish the governing equations. These effects are more pronounced in high Prandtl-number fluids, high permeable porous media and for large pressure gradients. It is noted that the effect of Brinkman term will only be in thin layers adjacent to rigid boundary, mainly within a distance $K^{1/2}$. Nield [6] confirmed that Brinkman model is applicable to porous media whose porosities are greater than 0.6. Vafai and Kim [7] addressed the work of Nield [8] and conveyed that the effect of porosity variation is not required for high porous medium, but it should be considered for dense porous medium. Walker and Homsy [9] studied the convective instabilities in an isotropic porous media. Later, Rees [10] analyzed the work of Walker and Homsy [9] in detail, by performing an asymptotic analysis. Barletta et al. [11] studied the convective instability in highly permeable porous medium, taking viscous dissipation into account.

The buoyancy force generated by heating at the bottom plate or an internal heat generation is one of the main sources of research interest for the onset of convection in a fluid-saturated porous medium. The onset of convection in a porous medium with internal heat generation is investigated in Gasser and Kazimi [12], in which the critical internal and external Rayleigh numbers were obtained for both stabilizing and destabilizing boundary conditions. Later, the finite amplitude convection in a porous layer heated from

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within was studied by Tveitereid [13] by analyzing steady solutions in the form of hexagons and two-dimensional rolls. Then after, Royer and Flores [14] studied natural convection in a rectangular heterogeneous anisotropic porous medium with an internal heat generation and compared numerical results of the problem with the cases of homogeneous and heterogeneous isotropic porous media. Subsequently, Parthiban and Patil [15] studied the thermal instability problem in an anisotropic porous media with internal heat source subjected to inclined gradients and obtained numerical results by using Galerkin technique. Nouri-Borujerdi et al. [16] studied the effect of uniform internal heat source in a horizontal porous layer with local thermal non equilibrium model, and then Nouri-Borujerdi et al. [17] focused on the influence of Darcy number on the onset of convection in the presence of uniform internal heat source where the equation of motion is modeled by Brinkman law.

The study of throughflow is important as it gives the possibility to control the potential convective instabilities by adjusting the throughflow. Wooding [18] was the first to study the Rayleigh instability of convection in a porous medium with throughflow. Convective flow in a two dimensional porous channel was studied by Sutton [19] when there is a net flow of fluid through the channel. Later, Homsy and Sherwood [20] provided the stability results for the problem of thermal convection for the case of net fluid discharge through porous medium, and obtained the linear and energy bounds. These bounds coincide at low discharge rates. The effect of throughflow on convection with inclined temperature gradients was addressed by Nield [21] and Qiao and Kaloni [22]. The onset of convection with internal heat generation and net throughflow (strong and weak) for different hydrodynamic boundary conditions is examined in the article by Khalili and Shivakumara [23]. The stability of a saline boundary layer formed by throughflow near surface of the porous medium was studied in Van Duijn et al. [24] and Pieters and Schuttelaars [25]. The linear instability of boundary layer formed by vertical throughflow in horizontal porous medium with local thermal nonequilibrium was examined in Patil and Rees [26]. The studies on the effect of vertical throughflow and internal heat source on the onset of convection in a layered porous medium were found in Kuznetsov and Nield [27], [28]. Later, the combined effects of throughflow and variable gravity on Hadley-Prats flow was studied by Deepika and Narayana [29]. The effect of vertical throughflow of a non-newtonian power-law fluid on the onset of convection in a horizontal fluid saturated porous layer was examined in Barletta and Storesletten [30]. In the absence of Brinkman term, the present problem is similar to the problem considered in Barletta et al. [31], in which viscous dissipation is considered as an internal heat source just as uniform internal heat source considered in this manuscript.

Owing to the important applications of the vertical throughflow and internal heat source in engineering applications, we analyze these combined effects in a fluid saturated Darcy-Brinkman porous layer. Section 2 deals with the governing equations and basic state solution, which is followed by the linear theory in Section 3 and the nonlinear theory via energy method in section 4. Numerical results of both the linear and nonlinear theories are discussed in Section 5. From the numerical results, we observed that the combination of vertical throughflow and internal heat source significantly effects the stability of the convection pattern. These observations are concluded in section 6.

## 2. Mathematical formulation

Let us consider a horizontal fluid-saturated porous layer $\Omega$ bounded by two planes separated at a distance $L>0$, such that $\Omega = \mathbb{R}^2 \times (-L/2, L/2)$ where $Ox\overline{y}z$ to be the Cartesian coordinate system with $\overline{x}, \overline{y}$ being the horizontal axes and $\overline{z}$ being the vertical axis. The Oberbeck-Boussinesq approximation is assumed to be valid throughout the domain i.e. the density $\rho$ is constant everywhere except in the body force term and it can be expressed as

$$\rho = \rho_0\left(1 - \beta\left(\overline{T} - \overline{T}_0\right)\right),$$

where $\rho_0$ is the density at reference temperature $\overline{T}_0$, $\overline{T}$ is the temperature, and $\beta$ is the thermal expansion coefficient.

The set of governing equations for the flow and heat transfer in dimensional form are given by

$$\overline{\nabla}\cdot\overline{\mathbf{u}} = 0, \tag{1a}$$

$$\frac{\mu}{K}\overline{\mathbf{u}} = -\overline{\nabla}\overline{P} + \mu_e\overline{\Delta}\overline{\mathbf{u}} + \rho g\widehat{\mathbf{k}}, \tag{1b}$$

$$(\rho h)_m\frac{\partial\overline{T}}{\partial\overline{t}} + (\rho h_p)_f(\overline{\mathbf{u}}\cdot\overline{\nabla})\overline{T} = k\overline{\Delta}\overline{T} + q, \tag{1c}$$

where $\overline{\mathbf{u}} = (\overline{u}, \overline{v}, \overline{w})$ is the seepage velocity vector, $\overline{\Delta} = \frac{\partial^2}{\partial\overline{x}^2} + \frac{\partial^2}{\partial\overline{y}^2} + \frac{\partial^2}{\partial\overline{z}^2}$, $\overline{P}$ is the pressure, $\widehat{\mathbf{k}} = (0,\ 0,\ 1)$ and $g$ is the acceleration due to gravity. Furthermore, $\mu, \mu_e, K, \overline{t}, \rho$ and $k$ are dynamic viscosity, effective viscosity, permeability, time, reference density of fluid and thermal conductivity, respectively. The term $q>0$ in (1c) is a (constant) internal heat source, and

$$(\rho_0 h)_m = (1-\phi)(\rho_0 h)_s + \phi(\rho_0 h_p)_f,$$

where $h_p$ is the specific heat of the fluid and $h$ is the specific heat of the solid, with the subscripts $f$, $s$ and $m$ referring to the fluid, solid and porous components respectively. The walls are assumed to be permeable, with a throughflow velocity $\overline{w}_0$ in the vertical direction such that

$$(\overline{u}, \overline{v}, \overline{w}) = (0, 0, \overline{w}_0), \quad \overline{T} = \overline{T_1}, \text{at} \quad \overline{z} = -\frac{L}{2}, \tag{1d}$$

$$(\overline{u}, \overline{v}, \overline{w}) = \left(0, 0, \overline{w_0}\right), \quad \overline{T} = \overline{T_0}, \text{at} \quad \overline{z} = \frac{L}{2}.$$

We define the following dimensionless variables

$$\mathbf{x} = (x, y, z) = \frac{1}{L}(\overline{x}, \overline{y}, \overline{z}) = \frac{\overline{\mathbf{x}}}{L}, \quad \mathbf{u} = (u, v, w) = \frac{L}{\alpha}(\overline{u}, \overline{v}, \overline{w}), \quad t = \frac{\alpha}{L^2 A}\overline{t},$$
$$A = \frac{(\rho h)_m}{(\rho h_p)_f}, \quad \alpha = \frac{k}{(\rho h_p)_f}, \quad P = \frac{K}{\alpha\mu}(\overline{P} + \rho_0 g\overline{z}), \quad T = \frac{\overline{T} - \overline{T}_0}{\overline{T}_1 - \overline{T}_0}, \tag{2}$$

where $\alpha$ is the thermal diffusivity, $A$ is the ratio of volumetric heat capacity $(\rho h)_m$ of fluid saturated porous medium to the volumetric heat capacity $(\rho h_p)_f$ of fluid. By substituting the dimensionless variables into (1a)–(1d), the following non-dimensional governing equations are obtained:

$$\nabla\cdot\mathbf{u} = 0, \tag{3a}$$

$$\mathbf{u} = -\nabla P + Da\,\Delta\mathbf{u} + Ra\widehat{\mathbf{k}}, \tag{3b}$$

$$\frac{\partial T}{\partial t} + (\mathbf{u}\cdot\nabla)T = \Delta T + Q, \tag{3c}$$