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Two-dimensional problem of thermoelectric materials with an elliptic hole or a rigid inclusion

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A B S T R A C T

The two-dimensional problems of an elliptic hole or a rigid inclusion embedded in a thermoelectric material subjected to uniform electric current density and energy flux at infinity are studied based on the complex variable method of Muskhelishvili and conformal mapping technique. The closed-form solutions of electric potential, temperature and stress components are presented according to electrical insulated and thermal exact boundary conditions on the rim of the hole or inclusion. Numerical results are carried out to illustrate the influence of the value of major to minor axis ratio of the elliptic geometry and heat conductivity of inhomogeneity on thermoelectric and stress fields. It is found that energy flux at surfaces of the hole or rigid inclusion does not vanish due to the Joule heat and Seebeck effect when the electric field is applied. In addition, stress induced by applied electric field has a non-linear relationship with the electric current density. The heat conductivity of the air inside the elliptic hole reduces the concentration factors of energy flux and stress. However, the concentration factors of energy flux and stress at the bonding interface increase with the increasing values of heat conductivity of the flat rigid inclusion.

1. Introduction

Thermoelectric materials for solid state energy conversion have been extensively used for electric cooling [1], electricity power generation in harvesting wasted heat [2], solar energy harvesting [3] and carbon reduction [4]. However, most thermoelectric materials are brittle in nature with low fracture strength and toughness [5]. Thermoelectric materials are subjected to significant stress induced by thermal gradient, thermal shock and externally applied mechanical loadings under in-service particularly when used in waste heat recovery [6]. Defects resulting from the manufacturing processes, such as cracks and holes, can cause electric potential and temperature discontinuities across the defects boundary and stress concentrations, then result in fracture [7–13]. Fracture problems in thermoelectric materials have been received much attention and investigated by many experts recently, like other structural materials subjected heat loadings problems [14–16]. The slow Vickers indentation crack growth behavior of Mg_2Si thermoelectric material was observed by Schmidt et al. [17]. Analytic solutions for the thermoelectric material containing a 2D crack problem were derived by Zhang and Wang [18], and Song et al. [19], respectively. It is found that electric current density, heat flux and stress exhibit traditional inverse square-root singularity at the crack front. Li et al. [20] examined the brittle failure behavior of $CoSb_3$ based skutterudite thermoelectric material by using large-scale molecular dynamics simulations. Based on the complex variable method, the study dealing with thermal stress concentration induced by an elliptic hole has been performed by Zhang and Wang [21]. In above-mentioned works, the boundary conditions on the hole or crack surfaces are always supposed to be electrical and thermal impermeable. The thermal conductivity of crack interior has great influences on the mode-II stress intensity factor for a thermal-medium crack in a thermoelastic material under a thermal loading [22]. Zhang and Wang [23] discussed the applicability of crack surfaces thermal boundary condition in a layered thermoelectric or metal/thermoelectric material, and the results show that the heat conductivity of air filled in a crack cannot be neglected.

On the other hand, in order to meet the various technology applications, much effort on thermoelectric materials has been devoted towards improving their figure of merit and energy conversion efficiency [24–29]. The efficiency of thermoelectric

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conversion depends on a dimensionless figure of merit $ZT = \varepsilon^2\gamma T/k$ [30], where γ, k, ε and T are the electric conductivity, heat conductivity, Seebeck coefficient and absolute temperature, respectively. Thermoelectric material with high energy conversion efficiency requires not only high Seebeck coefficient to have high voltage output, but also high electric conductivity to reduce Joule heat loss and low heat conductivity to maintain large temperature difference. However, it is rather difficult to increase ZT because of competing effects of electrical and thermal conductivities, that is, a good heat conductor is usually a good electric conductor as well. One of the main strategies in developing thermoelectric materials with high energy conversion efficiency is to engineer hybrid composites, especially with nanostructures. Comparing with the system with the single material, composite structures with the layered and/or inclusions have better thermal and mechanical behaviors [31–34]. The thermoelectric properties of semiconductor materials with nanoinclusions can be improved based on the concept of band bending at inclusion/semiconductor interfaces as an energy filter for electrons [35]. An effective path to increase the electrical conductivity while to decrease the thermal conductivity, and thus to improve the figure of merit by nano-inclusions was reported by Wang et al., [36]. Introduction of metallic inclusion can enhance the thermoelectric material performance of manganese silicide nanocomposites [37]. The presence of inclusions affects the energy conversion efficiency, strength and reliability of thermoelectric materials, and it is therefore important to study the local fields induced by the inclusions. Thermoelectric materials have increasing applications in the engineering, however, to our best knowledge relatively little work has been done for the inclusion problem based on the continuum mechanics theory.

In view of the above literature analysis, the purpose of this paper is to seek a general solution to the problem of a 2D thermoelectric material with an elliptic hole or rigid inclusion. The influence of heat conductivity of air inside the hole and inclusion on the distribution of electric field, temperature and stress is also investigated. The paper is organized as follows. Firstly, basic equations for thermoelectric material are outlined in Section 1. Next, closed-form solutions of the electric potential, temperature and stress fields for an elliptic hole and rigid inclusion are derived in Sections 3 and 4, respectively. Some numerical results are given in Section 5. Finally, Concluding remarks are made.

2. Basic equations for thermoelectric materials

We consider an isotropic thermoelectric material in which the electric potential is V and the absolute temperature is T. Such a material is characterized by the electric conductivity γ, thermal conductivity k and Seebeck coefficient ε. The governing equations for a thermoelectric material in the absence of electric charges and heat sources can be presented in the form [38,39],

$$\begin{aligned} \nabla\cdot\mathbf{j}_e &= 0 \\ \nabla\cdot\mathbf{q} + \mathbf{j}_e\cdot\nabla V &= 0 \end{aligned} \tag{1}$$

and the transport of electric current density vector $\mathbf{j}_e = [j_{ex}, \quad j_{ey}]^T$ and heat flux vector $\mathbf{q} = [q_x, \quad q_y]^T$ is given as

$$\begin{aligned} \mathbf{j}_e &= -\gamma\nabla V - \gamma\varepsilon\nabla T \\ \mathbf{q} &= -\gamma\varepsilon T\nabla V - \left(k + \gamma\varepsilon^2 T\right)\nabla T \end{aligned} \tag{2}$$

Notice that the uncoupled transport equations of electricity and heat are recovered when $\varepsilon = 0$, and the temperature function T and its gradient ∇T enter into the heat transport Eq. $(2)_2$, making the coupling nonlinear. Since energy is transported by both electrons and heat, the energy flux vector $\mathbf{j}_u = [j_{ux}, \quad j_{uy}]^T$ can be derived from the electric current density and heat flux as $\mathbf{j}_u = \mathbf{q} + \mathbf{j}_e V$. For the sake of convenience, an analytic function F is defined as $F = V + \varepsilon T$[18], then we have

$$\begin{aligned} \mathbf{j}_e &= -\gamma\nabla F \\ \mathbf{j}_u &= -\gamma F\nabla F - k\nabla T \end{aligned} \tag{3}$$

Combining Eqs. (1) and (3), the constitutive equations become,

$$\begin{aligned} \nabla^2 F &= 0 \\ k\nabla^2 T + \gamma(\nabla F)^2 &= 0 \end{aligned} \tag{4}$$

For the two-dimensional thermoelectric problem considered here, the general solutions of thermo-electro-elastic fields are derived detailed in our previous work [21], and summarized briefly as follows. The solutions of F and T can be expressed as,

$$\begin{aligned} F &= \mathrm{Re}[f_1(z)] \\ T &= \mathrm{Re}[g(z)] - \frac{\gamma}{4k} f_1(z)\overline{f_1(z)} \end{aligned} \tag{5}$$

where $z = x + iy, f_1(z)$ and $g(z)$ are unknown analytic functions and "Re" stands for the real part of a complex number. Re-writing Eq. (3) gives:

$$\begin{aligned} j_{ex} - ij_{ey} &= -\gamma f'_1(z) \\ j_{ux} - ij_{uy} &= -\frac{\gamma}{2} f_1(z) f'_1(z) - kg'(z) \end{aligned} \tag{6}$$

The boundary conditions of electric current density and energy flux are,

$$\begin{aligned} f_1(z) - \overline{f_1(z)} &= -\frac{2i}{\gamma}\int j_{en}(s)ds + \text{constant} \\ \mathrm{Im}\left[\frac{\gamma}{4} f_1{}^2(z) + kg(z)\right] &= -\int j_{un}(s)ds + \text{constant} \end{aligned} \tag{7}$$

where "Im" stands for the imaginary part of a complex number, $j_{en}(s)$ and $j_{un}(s)$ represent the normal electric current density and normal energy flux for the point s along the boundary, respectively.

The corresponding thermal stresses and displacement induced by the temperature field T can be obtained as [21,40]:

$$\begin{aligned} \sigma_{xx} + \sigma_{yy} &= 4\mathrm{Re}[\varphi'(z)] + \frac{2\mu\alpha^*\gamma}{k(\kappa+1)} f_1(z)\overline{f_1(z)} \\ \sigma_{yy} - \sigma_{xx} + 2i\sigma_{xy} &= 2\left[\bar{z}\varphi''(z) + \phi'(z)\right] + \frac{2\mu\alpha^*\gamma}{k(\kappa+1)} f_2(z)\overline{f'_1(z)} \end{aligned} \tag{8}$$

$$\begin{aligned} 2\mu\left[u_x + iu_y\right] = {} & \kappa\varphi(z) - z\overline{\varphi'(z)} - \overline{\phi(z)} + 2\mu\alpha^*\int g(z)dz \\ & - \frac{\mu\alpha^*\gamma}{k(\kappa+1)} f_2(z)\overline{f_1(z)} \end{aligned} \tag{9}$$

where $f_2{}'(z) = f_1(z)$, $\varphi(z)$ and $\phi(z)$ are complex stress potential functions to be determined, μ is the shear modulus, and κ and α^* are defined as follows,

$$\kappa = \begin{cases} \dfrac{3-\nu}{1+\nu}, & \text{for plane stress state} \\ 3 - 4\nu, & \text{for plane strain state} \end{cases} \tag{10}$$

$$\alpha^* = \begin{cases} \alpha, & \text{for plane stress state} \\ (1+\nu)\alpha, & \text{for plane strain state} \end{cases} \tag{11}$$

ν and α are the Poisson's ratio and the linear thermal expansion,