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A detailed comparative study on responses of four heat conduction models for an axisymmetric problem of coupled thermoelastic interactions inside a thick plate

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ABSTRACT

The objective of the present work is to analyze the thermoelastic interactions inside an infinitely extended thick plate due to an axisymmetric temperature distribution applied at the lower and upper surfaces of the plate under recent heat conduction models, namely Green-Naghdi-I model, Green-Naghdi-II model, dual phase-lag model and Green-Lindsay model. In order to investigate the problem under all these four heat conduction models simultaneously, we consider the basic governing equations under all these models and formulate our problem in a unified way. The potential function concept along with Laplace and Hankel transform techniques has been used to solve the problem. Inversion of Hankel transform has been carried out analytically to obtain the solution in Laplace transform domain. The short-time approximation technique is employed to invert the solutions obtained in Laplace transform domain and an appropriate analytical approach has been used to analyze the wave propagation and discontinuities of different wave fields. In addition, a numerical method has been used to invert the Laplace transform directly in order to find out the distributions of all the physical fields, like stress, temperature and displacement in the middle plane of the plate. Results are analyzed to make a comparative analysis of the predictions of Green-Naghdi-II model with the predictions by dual phase-lag thermoelastic model and other models. The special findings and differences among the predictions by four models have been highlighted.

1. Introduction

A widespread attention has been devoted to the generalization of the governing equations of coupled themoelasticity suggested by Biot [1]. The reason for the introduction of the theories of generalized themoelasticity is mainly due to the fact that the existing coupled thermoelasticity theory predicts infinite speed of propagation for thermal waves which is clearly not acceptable as this contradicts the physical observations that maximum wave speed can not exceed that of light in vacuum. In opposite to this conventional theory that is based on the parabolic type heat conduction equation predicting infinite speed of propagation for heat waves, the generalized theories are based on the hyperbolic type equation for thermal signals. It was found that in some situations, the classical theory gives wrong values for the temperature differing from values which are found in experiment. Specially, it has been realized that the heat equation obtained from Fourier's law of heat conduction fails to interpret the transient temperature field in the state containing short times, high frequencies, and small wavelengths [2]. Maurer and Thomson [3] showed that, by including a thin slab to a sharp thermal shock, its surface temperature is 300° C which is larger than the value predicted by the classical theory. Hence, the generalized theories of thermoelasticity are introduced to take into account of these unrealistic predictions of classical coupled theory. First of all, we would like to recall the generalized theory developed by Lord and Shulman [4] in which a new law of heat conduction model has replaced the Fourier's law with the inclusion of one thermal relaxation time parameter. Under this theory, the heat conduction equation is therefore reduced to a hyperbolic type equation which ensures the finite speeds of propagation for heat and elastic waves. Later on, Dhaliwal and Sherief [5] extended this theory to general anisotropic materials in the presence of heat source.

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Muller [6] firstly introduced the theory of generalized thermoelasiticty having two relaxation time parameters and later on, the more explicit version of this theory was introduced by Green and Lindsay [7] in which the temperature rates were considered among the constitutive variables. This theory also predicts finite speed of heat propagation as in Lord and Shulman's theory [4]. Later on, an alternative version of thermoelasticity have been proposed by Green and Naghdi [8–10] which is divided into three parts. They are subsequently being referred to as thermoelastic models of type Green-Naghdi model-I (GN-I), Green-Naghdi model-II (GN-II) and Green-Naghdi model-III (GN-III). The first two models are the subcases of type GN-III model. Temperature gradient and thermal displacement gradient are taken to be as constitutive variables in which the linearized version of model-I is closely related to classical thermoelastic model while there is no dissipation in model-II that occurs due to no change in internal energy and the internal rate of production of thermal entropy is assumed to be identically zero. This model is also termed as thermoelasticity without energy dissipation model. However, this does not happen in the general case i.e GN-III. The heat conduction equation under GN-III model is taken to be of the form

$$\vec{q}(p,\ t) = -\left[K\ \vec{\nabla}T(p,t) + K^*\ \vec{\nabla}\nu(p,t)\right],$$

where K and K^*, both being positive material parameters, are known as the thermal conductivity and conductivity rate, respectively and $\dot{\nu} = T$, where ν is termed as thermal displacement while $\vec{q}$ is the heat flux vector.

Further, one more model is introduced by Hetnarski and Ignaczak [11] which is subsequently known as low temperature thermoelastic model which is also explained by the system of non linear field equations in which the heat flux and free energy depend on temperature, strain tensor and heat flux. By focusing on the theoretical significance of various models, Hetnarski and Ignaczak [12] reviewed thoroughly in a survey article and domain of influence theorem is also explained by them with an initial value problem for Lord-Shulman [4], Green-Lindsay [7] and thermoelasticity without energy dissipation theories [10]. The dual phase-lag heat conduction model is proposed by Tzou [13] by taking into account the micro structural effects in heat transport process in which one parameter for heat flux vector and other for temperature gradient vector is used. The proposed heat conduction law is of the form

$$\vec{q}(p,\ t+\tau_q) = -\left[K\ \vec{\nabla}T(p,t+\tau_T)\right], \quad \tau_q > 0,\ \tau_T > 0,$$

where, τ_q and τ_T, are the delay time parameters. The micro structural interactions during heat transport phenomenon is captured by these two parameters where the micro structural effects like phonon scattering causes the delay times τ_T and τ_q caused by fast-transient effects of thermal inertia and above relation interprets that the gradient of temperature at a point p in the body at time $t + \tau_T$ corresponds to the heat flux vector at that point at time $t + \tau_q$. Clearly, above relation reduces to Fourier law in the case when $\tau_q = \tau_T = 0$. By assuming $\tau_q > 0$, $\tau_T > 0$, the dual phase-lag heat conduction model suggested by Tzou [13] has been further extended to a hyperbolic thermoelastic model with dual phase-lag effects by using its Taylor's series expansion and taking the second order term for $\vec{q}$ to ensure finite speed of thermal signals. Later on, the stability and uniqueness of solutions on this approximated two phase lag model is discussed in a detailed way by Quintanilla [14].

The main objective of the current work is to compare the results under the thermoelasticity without energy dissipation (Green-Naghdi-II model) for an axisymmetric temperature distribution applied at the upper and lower surfaces of an infinitely extended thick plate with the corresponding results predicted by the other heat conduction models like, Green-Naghdi model-I, dual phase-lag model and Green-Lindsay model. The problem in the context of Green-Lindsay model and dual phase-lag model has been studied by Aouadi [15] and Mukhopadhyay and Kumar [16]. In section 2 of the present work, the heat conduction equations under all four models are given which further have been written as a unified way and the governing equations involving the displacement, thermal and stress fields without any heat source or body force in isotropic medium are considered. In section 3, formulation of the problem has been carried out in which homogeneous, isotropic and an infinitely extended thick plate of thickness $2l$ is considered and the Helmholtz decomposition technique is used to decouple the problem. In section 4, the boundary conditions have been illustrated in which both the upper and lower planes of the plate have been taken as a traction free and these planes of the plate are subjected to an axisymmetric temperature distribution. In subsection 4.1, Laplace and Hankel transform techniques have been used to solve the problem and the solution has been found out in the transform domain. In subsection 4.2, inversion of Hankel transform has been carried out while in subsection 4.3, inversion of Laplace transform has been performed by using short time approximated method. On the basis of the short-time approximated results, discontinuities of physical fields are discussed by using the Boley's theorem [17] and in subsection 4.4, the analytical results have been explained. We further presented a detailed comparison of the results under GN-II model with the results in the context of other models. In section 5 the numerical computation is carried out by taking a copper material and the specific features of all the four models have been described. Section 6 represents an overall conclusion of the proposed work and investigates the prominent disagreement of the thermoelasticity without energy dissipation model with other models.

2. Governing equations

For homogeneous and isotropic elastic medium, the linearized heat conduction equations in the absence of any heat source in the contexts of different models can be written as

Green – Naghdi model – II (GN – II) [10]:

$$K^*\nabla^2 T = \left(\rho\ c_v \ddot{T} + \gamma\ T_0 \ddot{e}\right), \tag{1}$$

Green – Naghdi model – I (GN – I) [8]:

$$K\ \frac{\partial}{\partial t}\nabla^2 T = \left(\rho\ c_v \ddot{T} + \gamma\ T_0 \ddot{e}\right), \tag{2}$$

Dual phase – lag model [16]:

$$K\left(1 + \tau_T \frac{\partial}{\partial t}\right)\nabla^2 T = \left(1 + \tau_q \frac{\partial}{\partial t} + \frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2}\right)\left(\rho\ c_v \dot{T} + \gamma\ T_0 \dot{e}\right), \tag{3}$$

Green – Lindsay model [15]:

$$K\ \nabla^2 T = \rho\ c_v \left(1 + \tau_1 \frac{\partial}{\partial t}\right)\dot{T} + \gamma\ T_0 \dot{e}, \tag{4}$$

The governing equations involving the displacement, thermal and stress fields without any body force in an isotropic and homogeneous medium can be written as

The equation of motion:

$$\sigma_{ij,\ j} = \rho \ddot{u}_i, \tag{5}$$

Srain – displacement relation: