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Modeling of Joule heating and convective cooling in a thick-walled micro-tube

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ABSTRACT

The heating of a fluid in a metallic micro-tube can be realized at the inlet and/or within a certain section of micro-scale heat and fluid flow devices by using Joule heating which is a heat generation mechanism that occurs when an electric current is passed through the metallic wall. For the thermal analysis of fluid flow in an electrically heated micro-tube, the solution of conjugate heat transfer (to include effect of the axial conduction through the channel wall) together with Joule heating is required. An analytic solution is presented for conjugate heat transfer in an electrically-heated micro-tube in this study. The solution is obtained in the form of integrals by the method of Green's functions for the hydrodynamically fully-developed flow of a constant property fluid in a micro-tube. The current analytical model can predict the fluid temperature for a given wall thickness, wall material and applied voltage across the micro-tube. The effects of the wall thickness and the wall material on the normalized temperature distribution and the effectiveness parameter are discussed. The comparison of the normalized temperature for Joule heating and a spatially uniform heating is also presented.

1. Introduction

Micro-channels are the key component for many micro-scale fluid, heat and mass flow devices such as micro-heat-exchangers, micro-heat-sinks, micro-reactors and microfluidic fuel cells [1]. As the device scale moves to the micro-scale, some effects which can be safely neglected at the macro-scale have to be considered [2]. As a result of these effects, the classical fluid flow and heat transfer theories, correlations and design methodology may not be suitable at micro-scale. Additional effects depend on the working fluid (i.e. whether the flow is a gas or liquid flow), channel size as well as the channel material. Rarefaction and compressibility is an important parameter for gas flows even for low Mach number flows ($Ma \sim 0.1$) [3]. Depending on the degree of rarefaction, slip-flow and temperature-jump boundary condition together with thermal creep might need to be included in the analysis [4–6]. Viscous dissipation [7] and axial conduction within the fluid [8] may be quite important in the case of liquid flow. Moreover, in the case of liquid flow, the effect of the relative roughness [9] and the electro-viscous effects (if the channel wall is non-conducting) [10] may come into picture.

Although the channel wall thickness is usually small compared to the channel size for macro-channels, the thickness of the channel wall is often comparable to the channel size due to rigidity and fabrication concerns. Furthermore, although the convective heat transfer is the dominant heat transfer mode for macro-channels, the heat conduction through the channel walls and within the fluid becomes comparable with the fluid convection due to the low Peclet number nature of the heat transfer for micro-channels. Nowadays, micro-tubes made of copper, nickel, aluminum and stainless steel with an inner diameter of 100–900 μm are commercially available with different choice of connectors to build a fluidic network (the size data of some of the commercially available micro-tubes is given in Table 1). Typically, the wall thickness of micro-tubes is ranging between 25 and 700 μm. Especially for the small diameter micro-tubes, the wall thickness may be much larger than the inner diameter of the tube. Therefore, the heat transferred within the channel wall needs to be taken into account for the calculation of the overall heat transfer.

The conjugate heat transfer problem, which refers to the solution of convective heat transfer in a conduit together with heat conduction through conduit wall, has been studied by many researchers [11]. The conjugate heat transfer for macro-channels was

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Nomenclature

B_0	
G	Green's function ($s \cdot m^{-2}$)
g_{ave}	radius-averaged heating (W/m^3)
g_0	volume average heating (W/m^3)
h	heat transfer coefficient, ($W \cdot m^{-2} \cdot K^{-1}$)
j	imaginary number, $\sqrt{-1}$
k	thermal conductivity ($W \cdot m^{-1} \cdot K^{-1}$)
L	axial extent of heated region along the tube (m)
N	number of layers in fluid flow
Nu	Nusselt number, hr_f/k_f
Q	added heat flow via Joule heating (W)
q	heat flux ($W \cdot m^{-2}$)
q_0	equivalent heat flux ($Q/2\pi r_f L$)
Pe	Peclet number, Ur_f/α_f
r	radial coordinate (m)
r_f	radius of fluid flow (tube inner radius) (m)
r_w	outside radius of tube wall (m)
T	temperature (K)
T_0	Far-upstream temperature (K)
u	local velocity (m/s)
U	average velocity (m/s)
x	axial coordinate (m)

Greek

α	thermal diffusivity ($m^2 \cdot s^{-1}$)
β	wave number (m^{-1})
δ	Dirac delta function
ε	Heat exchanger effectiveness
ν	modified wave number

Superscripts

+	dimensionless quantity
−	spatial Fourier transform

Subscripts

f	fluid
w	wall
i	within layer i
m	mean-flow value

Table 1
Some typical values for the size and material of the microtubes.

Reference	Material	d (μm)	D (μm)	Wall thickness (μm)
[25,26]	Stainless steel	170	1588	709
[25,26]	Stainless steel	510	1588	539
[25,26]	Stainless steel	750	1588	419
[27]	Nickel	200	780	290
[27]	Nickel	300	350	25
[27]	Nickel	350	600	125
[27]	Nickel	500	800	150

studied couple decades ago with the exclusion of the axial conduction within the fluid [12,13]. More recently, parallel to the development in the micro-fabrication techniques, the conjugate heat transfer for micro-channels was investigated for different geometries such as circular [14–16], parallel-plate [17–20], rectangular [21–23] and converging-diverging [24]. In these studies, the axial conduction within the fluid is also considered due to the low Peclet number nature of the flow. Inclusion of axial heat conduction in the channel wall and in the fluid introduce some challenges in the solution. One challenge is the coupling of the heat conduction problem with the heat convection problem associated with the inclusion of the axial heat conduction in the wall. A second challenge is the non-self adjoint eigen-value nature of the problem if a solution is an analytical technique based-on eigen-function expansion associated with the inclusion of the heat conduction within the fluid [8]. Many researchers implemented numerical solution to overcome these drawbacks [14–16,20–22,24]. Alternatively, many researchers proposed different analytical techniques which offers a fast and highly accurate solution, namely infinite Fourier transform [18,19] and general integral transform technique [23].

The heating of a fluid in a micro-tube can be desirable at the inlet and/or within a certain section of micro-scale heat and fluid flow devices. For macro-tubes, the convenient way to supply heat to the channel wall is to wrap an electric-resistance heater around a tube which realize a constant heat flux thermal boundary condition at the tube boundary. However, in the case of metallic micro-tubes, the convenient way to supply heat to the channel wall is to use Joule heating, which is a heat generation mechanism that occurs when an electric current is passed through the metallic wall. In this case, the heat flux at the solid-fluid interface is a result of the volumetric heat generation which takes place within the channel wall. Several researchers utilized electrical heating of a tube for the experimental investigation of the heat transfer through micro-tubes [25,26,28]. For the thermal analysis of fluid flow in an electrically heated micro-tube, the solution of conjugate heat transfer (to include effect of the axial conduction through the channel wall) together with Joule heating is required. To properly predict the Joule heating, the electric field within the tube wall has to be obtained.

1.1. Present study

An analytic solution is presented for a steady-state conjugate heat transfer in an electrically-heated micro-tube which is a generalized version of the classical Graetz problem. The solution is obtained in the form of integrals by the method of Green's functions for the hydrodynamically fully-developed flow of a constant property fluid in a micro-tube. The effect of the conjugate heat transfer together with the internal energy generation within the micro-tube wall due to Joule heating is analyzed. The current analytical model can predict the fluid temperature for a given wall thickness, wall material and applied voltage across the micro-tube. The effects of the wall thickness and the wall material on the normalized temperature distribution and the effectiveness parameter are discussed. The comparison of the normalized temperature for Joule heating and a spatially uniform heating is also presented. Such a model is beneficial for the researchers and/or engineers who would like to thermally condition fluid in a micro-tube which can easily be realized as the pre-conditioner of a micro-reactor which would handle different species. To the best of authors' knowledge, an analytical model for such a multi-physics problem has not been developed for the analysis of heat transfer in a micro-tube.