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# Three dimensional heat transfer from a square cylinder at low Reynolds numbers

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ABSTRACT

Three-dimensional unsteady flow and heat transfer from an isothermal square cylinder subjected to crossflow of air is numerically investigated. The governing continuity, momentum and energy equations are solved using implicit fractional step method. The first order spatial derivatives are discretized using a third-order upwind scheme while the second order derivatives from the viscous terms are discretized by central-difference formula. The momentum equations are solved separately using Crank-Nicholson time-stepping method. The Poisson type equations are solved using Jacobi method with Chebyshev acceleration procedure.

The heat transfer characteristics of a square cylinder subjected cross flow of air and the three-dimensionality of the flow and its effects are assessed. Three dimensional instantaneous iso-temperature surfaces and in the wake region and local Nusselt number variations on the cylinder surfaces along z-axis were provided. The time histories of the Nusselt numbers at the cylinder surfaces are obtained. Also instantaneous and mean deviation of the local Nusselt numbers on the cylinder surfaces both in spanwise direction and x-y plane are provided for Re = 185 and 250 which are representative of typical A-mode and B-mode vortex structures in spanwise direction respectively.



## 1. Introduction

The fluid flow and heat transfer around a bluff body has received increasing interest in many engineering fields due to its applications such as heat exchangers, cooling systems, cooling of electronic systems, pipelines, flow around nuclear rods, heat lost from buildings, etc. Therefore, flow around the unconfined circular and/or square cylinders has been experimentally and numerically studied for the last decades.

It has been long established that the fluid flow around a cylinder, at small Reynolds numbers ($Re < 165$), is laminar and two-dimensional. At relatively large Reynolds numbers ($Re > 165$), the flow becomes three-dimensional having well-known discontinuities at the St-Re curve. This discontinuity was reported by Williamson [1–3], Luo et al. [4] and others.

The transition phenomena in the wake of a circular cylinder were studied in detail by Luo et al. [4]. Mode A and mode B type instabilities in the wake of a square cylinder were studied and determined by identifying the discontinuities in the St-Re curves. The authors found that Reynolds number had mean values of 160 and 204 for the onset of the mode A and B type instabilities, respectively. The experimental study of Luo et al. [4] also showed that the aspect ratio of the cylinder and the end plates which used to promote a three-dimensional flow have a strong influence on the wake of a cylinder. They obtained the St-Re discontinuity by resolving the flow for sufficiently small increments of the Reynolds number. It was shown that when the cylinder aspect ratio was less than 22.2, the magnitude of the Strouhal number increases with the cylinder aspect ratio and remains almost constant for larger aspect ratios.

Large scale vortex shedding in the wake region and the existence of discontinuities in the St-Re relationships was studied by Williamson [1–3]. It was found that both of these wake characteristics are directly related to each other. The St discontinuities are caused by a transition from one oblique shedding mode to another oblique mode. Vortex dislocation is observed due to vortices in neighboring cells which move out of phase with each other. However, by manipulating the spanwise end conditions of the cylinder, it becomes possible to induce parallel vortex shedding which results in continuous St-Re curve.

Luo et al. [5] experimentally investigated the wake transition

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regime of a square cylinder. The authors did not observe discontinuities on St-Re curve even though they identified type A and type B vortices in the spanwise region while reporting. The transition to mode A an B type instabilities, which had a spanwise length of 5.2D and 1.2D, took place at Re~160 and 200, respectively.

Saha et al. [6] numerically studied the special evolution of vortices and transition to three-dimensionality in the wake of a square cylinder. They reported that three dimensionality takes place between 150 < Re < 175 and secondary vortices of mode A persist at 175 < Re < 240. B type vortices present in the wake region at about Re~250.

Robichaux et al. [7] investigated the onset of three-dimensionality in the wake of a square cylinder by applying Floquet stability analysis. They reported a long-wavelength (mode A) three-dimensional instabilities appears first at Re~161, followed by short-wavelength (mode B) instabilities at Re~190. In addition to these two modes, they observed intermediate-wavelength mode at Re~200. Their numerical analysis did not present discontinuity at the St-Re curve.

In addition to the flow characteristics, two-dimensional forced convection heat transfer from a cylinder also was investigated by numerous researchers. One of these studies is performed by Sharma and Eswaran [8]. They investigated the flow structure and heat transfer characteristics of an isolated cylinder in the 2-D laminar flow regime for $1 \leq \mathrm{Re} = 160$ and $\mathrm{Pr} = 0.7$. They reported that the mean Nusselt number changed along the cylinder surface. The maximum Nusselt number is obtained at the front surface while it takes intermediate values at the top and bottom faces. The authors also proposed a heat transfer correlation applicable for 2-D flow regime. Recent study, on the flow and heat transfer across a circular cylinder has been performed by Golani and Dhimant [9] for $\mathrm{Re} = 50–180$ and $\mathrm{Pr} = 0.7$. They obtained variation of average Nu with Re.

Karanth et al. [10] studied the effects of cylinder oscillation on the average Nusselt number. The heat transfer from the oscillating cylinder increased with the increasing velocity amplitude. The location of maximum local Nusselt number also oscillated between the upper and lower surface of the cylinder in the case of transverse oscillation.

Two-dimensional laminar air flow (between $0.001 \leq \mathrm{Re} = 170$) and heat transfer past a circular isothermal cylinder was investigated by Shi et al. [11] with an emphasis on the heating effects on the flow characteristics. The numerical experiments with temperature dependent fluid properties resulted in depicting the effects of dynamic viscosity and density variations on the vortex shedding frequency.

The effect of blockage on heat transfer from a square cylinder in a channel was studied by Turki et al. [12]. It was found that the flow was unstable when Richardson number crosses the critical value of 0.13. The heat transfer coefficient was influenced by thermally induced flow when $\mathrm{Ri} > 0.2$. A more comprehensive investigation of the heating effects on the vortex structure in the wake was performed by Ren et al. [13], Van Steenhoven and Rindt [14], Badr [15] Van Steenhoven and Rindt [14] investigated 2-D behavior and 3-D transition behind a heated circular cylinder for $\mathrm{Re} = 100$. It was reported that the vortex street had undergone a negative deflection which caused by difference between the strengths of the upper and lower vortices at Ri<1. For Ri>1, an early 3-D transition with mushroom-type structure appear on top of the upper vortex row. The three-dimensional transition of a water flow around a heated circular cylinder was studied by Ren et al. [13]. Escaping mushroom-type structure in the far wake was observed for $\mathrm{Re} = 85$, $\mathrm{Pr} = 7$ and $\mathrm{Ri} = 1$ case. The origin of the type structure was the generation of streamwise vortices in the near wake.

Persillon and Braza [16] showed that the frequency discontinuities in Re-frequency curve are associated with a discontinuity in the local kinetic energy distribution in the near-wake region. Zhank et al. [17] observed four different physical instabilities namely vortex adhesion mode and three near-wake instabilities which are associated with three different spanwise wave-length. Although many experimental studies performed on the discontinuities of the St-Re curves, the foregoing numerical studies focused on investigation of the long-wavelength, short-wavelength vortex structure in the spanwise direction of wake region, and variation of St number with the Re number.

Although 3-D flow characteristics passed a cylinder at low Reynolds numbers have been extensively studied both experimentally and numerically, the numerical studies involving heat transfer from cylinders are based on 2-D simulations. The effects of the 3-D flow on the heat transfer characteristics have yet to be investigated.

The aim of the present study is to determine the heat transfer characteristics from the cylinder in cross flow of air and explore the effects of the three dimensional flow. For this purpose, 3-D simulations were performed for $\mathrm{Re} = 155–250$ including transformation from 2-D flow and heat transfer to 3-D. The time histories of the mean Nusselt number evaluated over the cylinder surface area are computed. Instantaneous and mean deviation of the local Nusselt numbers on the cylinder surfaces both in spanwise direction and x-y plane are also provided.

## 2. Governing equations and boundary conditions

Dimensionless form of the unsteady Navier-Stokes equations for incompressible three-dimensional fluid flow are as follows:

$$\frac{\partial u_i}{\partial x_i} = 0 \tag{1}$$

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{1}{\mathrm{Re}} \frac{\partial^2 u_i}{\partial x_j \partial x_j} \tag{2}$$

$$\frac{\partial \theta}{\partial t} + \frac{\partial (u_j \theta)}{\partial x_j} = \frac{1}{\mathrm{RePr}} \frac{\partial^2 \theta}{\partial x_j \partial x_j} \tag{3}$$

where $u_i$, are velocity components, $p$ is the pressure, and $t$ is the dimensionless time. The dimensionless temperature $\theta$ is defined as $\theta = (T - T_\infty)/(T_\omega - T_\infty)$. The other dimensionless parameters governing the flow and heat transfer parameter defined as $\mathrm{Re} = U_\infty D/\nu$ and $\mathrm{Pr} = \mu c_p/k$ where D is the side lenght of the square cylinder and $U_\infty$ is the free stream velocity at the inlet. The Prandtl number of air is assumed to be 0.7. The geometry and dimensions considered in this simulation is illustrated in Fig. 1(a) and (b). A fixed square-cylinder ($D \times D \times 6D$) is maintained at a constant temperature $T_w$, and it is placed in a free stream of velocity U and temperature $T_\infty$. The boundary conditions are as follows:

For the inlet, top and bottom boundaries, $u = 1,\ v = 0,\ \theta = 0$,

For the top and bottom boundaries, $v = \frac{\partial u}{\partial y} = \frac{\partial w}{\partial y} = \frac{\partial \theta}{\partial y} = 0$,

For the outlet, $\frac{\partial \varphi}{\partial t} + u_c \frac{\partial \varphi}{\partial x} = 0$,

On the cylinders walls, $u = v = w = 0$ (no slip).

On the cylinder, $\theta_\omega = 1$ (isothermal wall).where $\varphi$ is all velocity components and temperature, $u_c$ is the velocity of the vortices leaving the outflow plane. The lift and drag coefficients of downstream cylinder are computed from