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Nonlinear dynamics of two-layer systems with a temperature-dependent heat release

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A B S T R A C T

Nonlinear convective flows developed under the joint action of buoyant and thermocapillary effects in a two-layer system with rigid heat-insulated lateral walls, have been investigated. The influence of a temperature-dependent interfacial heat release/consumption on oscillatory regimes under the action of an inclined temperature gradient, has been studied. It is shown that sufficiently strong temperature dependence of interfacial heat sources and heat sinks can lead to the change of the sequence of bifurcations and the development of new nonlinear regimes. Specifically, the period doubling bifurcations have been obtained.

1. Introduction

Convective phenomena in systems with interfaces have been a subject of an extensive investigation at the past few decades (for a review, see Refs. [1,2]). Traditional fields of application of the interfacial convection are chemical engineering [3] and materials processing [4].

It is known that the stability problem for the mechanical equilibrium in a system with an interface is not self-adjoint (see, e.g. [1,5]), thus an oscillatory instability is possible. The hydrodynamic and thermal interaction between convective motions on both sides of the interface can produce oscillations. The mechanism of oscillatory instability was explained in Ref. [6] for the transformer oil - formic acid system. However, the minimum value of the Grashof number in Ref. [6] was achieved on the monotonic mode. Further, some artificial systems with an oscillatory instability have been suggested in Ref. [7] (see also [1]) and [5]. A weakly nonlinear theory of oscillations for model systems has been described in Refs. [8,9].

In reality, the stability of the mechanical equilibrium is determined by the joint action of the buoyancy effect and the thermocapillary effect. When the system is heated from below, the competition between both effects can lead to the development of the specific type of oscillations. This phenomenon was first discovered in Ref. [10] (see also [1] [11]). Oscillations appearing at the instability threshold have been observed in experiments of Degen et al. (see Ref. [12]).

Under experimental conditions, the temperature gradient is not perfectly vertical and the horizontal component of the temperature gradient appears. The appearance of this component changes the situation completely: at any small values of the Marangoni number ($M \neq 0$), the mechanical equilibrium becomes impossible, and a convective flow takes place in the system. Thus, it is reasonable to consider the influence of the horizontal component of the temperature gradient on convective regimes developed in the system (see Refs. [13,14]). The Marangoni convection under the action of an inclined temperature gradient in two-layer systems, where the instability of a thermocapillary flow manifests itself in the form of hydrothermal waves or convective patterns, has been investigated in Ref. [15]. The influence of the horizontal component of the temperature gradient on convective oscillations in a two-layer system filling a closed cavity with *rigid heat-insulated lateral walls*, has been considered in Ref. [16]. It was shown that the horizontal component of the temperature gradient could lead to the violation of the symmetry conditions and the appearance of asymmetric oscillatory flows.

The interaction between the convection caused by the temperature gradient directed perpendicularly to the interface and the convective flow produced by the horizontal component of the

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temperature gradient in *a laterally infinite two-layer system* in the presence of buoyant and thermocapillary effects, has been studied in Ref. [17].

There are various physical phenomena that can be the origin of a heat release on the interface. For example, the interfacial heat release accompanies an interfacial chemical reaction (see, e.g. [18]) and heat consumption accompanies the evaporation [19]. The interfacial heating may be generated, e.g., by an infrared light source. The infrared absorption bands of silicone fluids and water are essentially different [20], therefore the light frequency can be chosen in a way that one of fluids is transparent, while the characteristic length of the light absorption in another liquid is short.

The presence of a constant, spatially uniform heat release at the interface can lead to the appearance of an oscillatory instability [21]. The influence of a *constant* interfacial heat release and heat consumption on convective oscillations has been studied in the case of *rigid heat-insulated lateral walls*, corresponding to a closed cavity in Refs. [22] [23], and in the case of *periodic boundary conditions on the lateral walls*, corresponding to a laterally infinite two-layer system in Ref. [24]. In Refs. [22] and [24], the temperature gradients were directed *perpendicularly* to the interface in both layers. The action of an inclined temperature gradient on nonlinear flows has been investigated in Ref. [23].

In reality, the interfacial heat release/consumption is not constant but is determined by the interfacial temperature. The heat produced on the interface is equal to the heat transported to the rigid boundaries of the system. The influence of a temperature-dependent interfacial heat release/consumption on convective regimes in a two-layer system with *periodic boundary conditions on the lateral walls*, has been studied in Ref. [25]. The temperature gradients in both layers were directed *perpendicularly* to the interface.

In the present paper, that can be considered as an extension of [23], we study *the influence of a temperature-dependent interfacial heat release/consumption* on nonlinear oscillatory convective regimes in a two-layer system with *rigid heat-insulated lateral walls* under the action of an *inclined temperature gradient*. The wide range of parameter Q_T, characterizing the temperature dependence of heat sources and heat sinks at the interface, has been considered. It is shown that sufficiently large values of $|Q_T|$ can lead to the change of the sequence of bifurcations and the development of new nonlinear regimes in the system.

The paper is organized as follows. The mathematical formulation of the problem in a two-layer system is presented in Section 2. The nonlinear approach is described in Section 3. Nonlinear simulations of the finite-amplitude convective regimes are considered in Section 4. Section 5 contains some concluding remarks.

2. Formulation of the problem

We consider a system of two horizontal layers of immiscible viscous fluids with different physical properties (see Fig. 1). The variables referring to the top layer are marked by subscript 1, and the variables referring to the bottom layer are marked by subscript 2. The system is bounded from above and from below by two rigid plates, $z = a_1$ and $z = -a_2$. The temperature on the horizontal plates is fixed in the following way: $T_1(x, a_1) = -A_h x$, $T_2(x, -a_2) = -A_h x + \theta, A_h > 0$, where θ is the total temperature drop between the plates.

A temperature-dependent interfacial heat release $Q(T_\Gamma)$ is determined as follows:

$$Q(T_\Gamma) = Q\left(T_\Gamma^0\right) + Q'\left(T_\Gamma^0\right)\left(T_\Gamma - T_\Gamma^0\right),$$

where T_Γ is the actual temperature on the interface, and T_Γ^0 is the interfacial temperature at the mechanical equilibrium state. Here

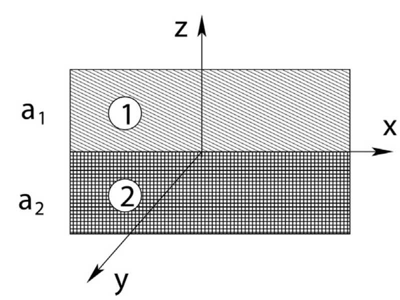

Fig. 1. Geometrical configuration of the two-layer system and coordinate axes.

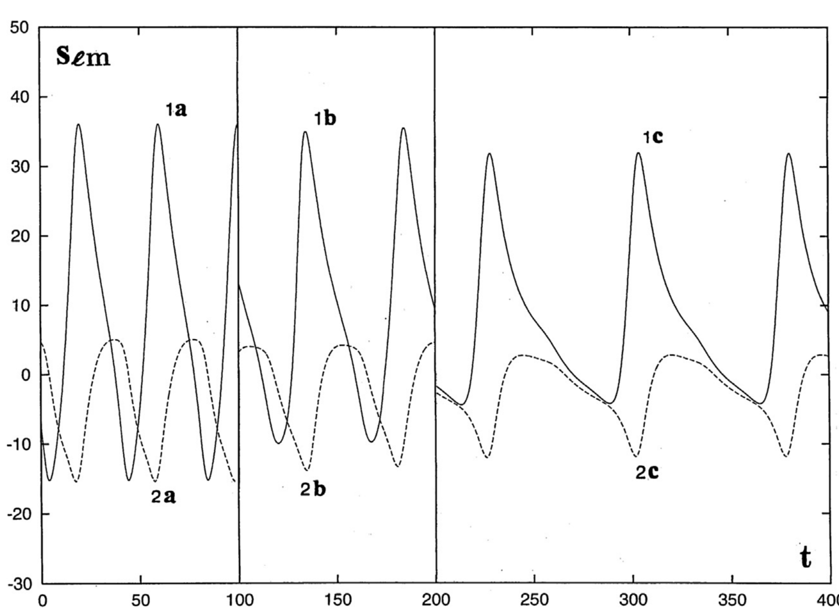

Fig. 2. Dependences of $S_{l,m}$ on time $(m = 1, 2)$ at $Q_T = 0$ (lines 1a, 2a); −0.75 (lines 1b, 2b); −1.35 (lines 1c, 2c); $G = 112.5$; $K = 0.025$; $\varepsilon = 0.01$; $G_Q = -25$; $L = 2.74$; $a = 1$.