Available online at www.sciencedirect.com
ScienceDirect
Comput. Methods Appl. Mech. Engrg. 197 (2008) 1280–1295
Computer methods in applied mechanics and engineering
www.elsevier.com/locate/cma

Consistent integration of the constitutive equations of Gurson materials under adiabatic conditions

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Received 17 April 2007; received in revised form 29 October 2007; accepted 5 November 2007
Available online 22 November 2007

Abstract

In this paper, a consistent integration procedure for the thermoviscoplastic version of the complete Gurson model is proposed. With adiabatic conditions considered and with the use of the backward Euler integration scheme, a numerical algorithm implicit in all variables as well as the corresponding algorithmic operator have been developed. The proposed algorithm was implemented in a finite element code and its performance is demonstrated with the numerical simulation of different examples.

Keywords: Gurson model; Thermoviscoplasticity; Consistent integration

1. Introduction

It is well known that the mechanical properties of an alloy will change under different strain-rate loadings. Thus, an understanding of the constitutive behaviour of metals over a wide range of strain rates is of importance in many advanced processes in engineering, such as metal forming [1], machining [2,3], structures under crashes [4] and high-speed impact on metallic armours [5,6]. The strain-rate sensitivity, defined as the amount of change of flow stress because of a change in strain rate, greatly helps to resist instabilities, and thus should be considered in all these processes. Viscoplasticity, as a branch of the theory of solid mechanics, analyses the effect of strain rate in the inelastic properties of the material. A widely used viscoplastic formulation is the *overstress* model (such as Perzyna [7] and Duvaut-Lions [8]). The main feature of overstress models is that the rate-independent yield function can become larger than zero, allowing excursions of stress states outside of the static yield surface. With the use of the overstress models, the consistency condition is not fulfilled and the Kuhn–Tucker conditions are not applicable.

A second approach to describe viscoplastic effects is referred to as the *consistency* model, first proposed by Wang et al. [9] and used by other authors (Ristinmaa and Ottosen [10], Winnicki et al. [11] and Heeres [12]). In the consistency approach, a rate-dependent yield surface is employed and it can expand and shrink not only by softening or hardening effects, but also by softening/hardening rate effects, i.e.,

$$f(\boldsymbol{\sigma}, \boldsymbol{\kappa}, \dot{\boldsymbol{\kappa}}) = 0 \quad \text{at } \dot{\lambda} > 0 \tag{1}$$

with $\boldsymbol{\kappa}$ being a vector including all the state variables and λ the plastic multiplier. The standard Kuhn–Tucker conditions for loading and unloading remain valid when using this formulation. Furthermore, the consistency model yields numerical algorithms with a somewhat higher convergence rate than that derived by the overstress model [11,13].

The above-mentioned processes involving high strain rates are often accompanied by a rise in temperature due to the dissipation of plastic work. This means that the energy-balance equation governing temperature evolution should involve terms arising from a thermomechanical

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0045-7825/$ - see front matter
doi:10.1016/j.cma.2007.11.008

coupling. This effect cannot be avoided in most cases in finite deformation problems, especially when the material is heated rapidly and the amount of local heat transfer decreases due to limited thermal diffusion. The thermodynamic process deviates from the isothermal conditions and approaches adiabaticity, leading to large variations in the temperature field. Calculations should then use temperature-variable mechanical properties, and thermal softening of the material should be considered since dynamic plastic instabilities, such as adiabatic shear bands or necking, are commonly triggered by a decrease in the yield limit with increasing temperatures [14–17]. Numerical investigations in thermoviscoplasticity are frequently related to overstress models [18,19]. Recently, Zaera and Fernández-Sáez [20] have extended the consistency model to analyse the thermoviscoplastic behaviour of metals. They have proposed a fully implicit scheme which is easily implemented and inherits the robustness and stability of return mapping algorithms.

The above works do not take into account the micromechanics phenomena responsible for damage and fracture when ductile metals are used. In these cases, processes can be accompanied by major accumulated damage, causing internal deterioration and macroscopic failure. In most ductile metals, the damage mechanisms involve the nucleation of voids at second particles, by decohesion of the particle–matrix interface or by particle fracture, the further growth of voids due to the plastic straining of the surrounding matrix and, finally, the coalescence to form a macroscopic crack.

In the continuum-mechanics framework, the most widely used model to describe the aforementioned processes was originally developed by Gurson [21] and further improved by Tvergaard [22,23] and Tvergaard and Needleman [24] (the so-called GTN model). This model predicts the coalescence of voids when a critical volume fraction of them, empirically selected, is reached. Following the plastic-limit-load approach proposed by Thomason [25] a new criterion for void coalescence has been incorporated into the GTN model (called the complete Gurson model [26–28]). This complete Gurson model considers the critical volume fraction of the void as a material- and stress-dependent parameter.

Different authors have applied the Gurson model to dynamic problems, involving inertial and high-strain-rate effects [29–35]. Recently, Betegón et al. [36] included strain-rate effects in the consistency model of Gurson materials and they have proposed an implicit method to integrate the constitutive equations of ductile metallic materials under high strain rates based on the complete Gurson model.

In the above-mentioned works, the thermal effects accompanying the deformation processes at high strain rates are not taken into account. Srikanth and Zabaras [37] proposed a thermoplastic model coupled with ductile damage using the GTN approach to analyse metal forming processes. Thermoviscoplastic versions of the Gurson model, including strain-rate and temperature effects, has been developed by Koppenhoefer and Dodds [38], Eberle et al. [39], Needleman and Tvergaard [40], Tvergaard and Needleman [41], and Hao and Brock [42].

In the present paper, thermal effects are included in the consistency viscoplastic model of void-containing materials modelled by the Gurson constitutive equations, and a consistent integration procedure for the thermoviscoplastic version of the complete Gurson model is proposed. With the consideration of adiabatic conditions and with the use of the backward Euler integration scheme, a numerical algorithm implicit in all variables as well as the corresponding algorithmic operator was developed. The proposed algorithm was implemented in the finite element commercial codes ABAQUS/Standard [43] and ABAQUS/Explicit [44] through the material subroutines *UMAT* and *VUMAT*, respectively, and its performance is demonstrated with the numerical simulation of different examples.

2. A themoviscoplastic version of the complete Gurson model

2.1. Basic kinematics

Let $\mathscr{B}_t \subset \mathbb{R}^3$ define the current configuration at time $t \in \mathbb{R}$ of a continuum body $\mathscr{B}$, and $\mathscr{B}_0 \subset \mathbb{R}^3$ the reference, initial or undeformed configuration at time $t = 0$ (considered coincident). Let $X \in \mathscr{B}$ be a particle in the body, $\mathbf{X} \in \mathscr{B}_0$ the position of X in the reference configuration $\mathscr{B}_0$, and $\mathbf{x} \in \mathscr{B}_t$ the position of X in the current configuration $\mathscr{B}_t$. The motion of the body is defined by a smooth time-dependent mapping $\varphi_t : \mathscr{B}_0 \to \mathscr{B}_t$, that is $\mathbf{x} = \varphi_t(\mathbf{X})$. The deformation gradient $\mathbf{F}$ is a two-point tensor defined by

$$\mathbf{F} = \nabla_{\mathbf{X}} \varphi_t(\mathbf{X}) = \frac{\partial \mathbf{x}}{\partial \mathbf{X}}. \tag{2}$$

This tensor transforms an infinitesimal material vector $\mathrm{d}\mathbf{X} \in \mathscr{B}_0$ into the corresponding spatial vector $\mathrm{d}\mathbf{x} \in \mathscr{B}_t$:

$$\mathrm{d}\mathbf{x} = \mathbf{F}\,\mathrm{d}\mathbf{X}. \tag{3}$$

The application of the theorem of polar decomposition to $\mathbf{F}$ implies:

$$\mathbf{F} = \mathbf{R}\mathbf{U} = \mathbf{V}\mathbf{R}, \tag{4}$$

$\mathbf{R}$ being the polar orthogonal rotation tensor, $\mathbf{U}$ the material or right stretch tensor and $\mathbf{V}$ the spatial or left stretch tensor. The velocity of a particle $\mathbf{v}_t$ at time t is defined consistently as the time derivative of the spatial position $\mathbf{x}$:

$$\mathbf{v}_t(\mathbf{x}) = \frac{\partial \varphi_t^{-1}(\mathbf{x})}{\partial t}. \tag{5}$$

The velocity gradient tensor $\mathbf{l}$ is the spatial derivative of $\mathbf{v}_t$, which is given by

$$\mathbf{l} = \nabla_{\mathbf{x}} \mathbf{v}_t = \frac{\partial \mathbf{v}_t(\mathbf{x})}{\partial \mathbf{x}} = \dot{\mathbf{F}}\mathbf{F}^{-1}. \tag{6}$$

The symmetric and skew-symmetric parts of the latter expression supply two additional rate tensors: the rate of deformation tensor $\mathbf{d}$ and the spin tensor $\mathbf{w}$: