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# Transition to turbulent heat transfer in heated vertical channel - Scaling analysis

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ABSTRACT

A thermally driven flow in a vertical channel with heated walls is studied through a scaling analysis and results are compared with the experimental data reported in [1]. As the geometry is symmetrical with respect to the vertical axis, the half-channel is considered. The domain is split into seven zones in which the flow behaviour results from a balance between several phenomena. This zonal analysis exhibits the main phenomena that drive the flow in each zone. The study leads to an expression of the wall temperature profile below and above the transition. An expression is given for the driving pressure that is found to increase along the channel. The increase rate is much more important above the transition due to the fluid heating in the center of the channel. Finally, The maximum Reynolds stress is expressed in terms of the mean velocity values and is found to fit well with the experimental data.



## 1. Introduction

Natural convection in vertical channels has attracted a large numbers of researches due to its applications to cooling system of electronic components or to thermally driven energy components in buildings. For example, photovoltaic double-façades are composed of a vertical photovoltaic wall separated from the primary envelope of the building by an opened air gap where natural convection occurs. Understanding heat transfer inside the double-façade is a challenge for energy management in the building. Such a complex component can be modelled by a vertical channel heated on one wall, the other one being adiabatic and located in an isothermal and infinite surrounding. Two important characteristics of such a configuration have to be noticed. Firstly, this flow is characterized by vertical isothermal lines near the heated wall that are parallel to the direction of the buoyant force. Therefore, there is no zero-velocity solution for the flow. Secondly, this open flow is characterized by a mean flow rate, the prediction of which is of great interest for engineering applications.

An experimental study carried out by the authors on a similar configuration characterized by a vertical channel in water with symmetrical heating, has been first reported in Ref. [2]. This study has led to a more detailed experimental characterization reported in Ref. [1] which is also referred in this article as the companion paper or the experimental study. The aim of the present study is to provide a scaling analysis of this experimental study. Indeed, numerous experimental or numerical data for such configuration are given in the scientific literature (see Refs. [1,2], and references therein) but few of them deal with scaling analysis whereas it is a powerful tool to understand physical phenomena that drive a flow. Indeed, such analysis have already been used to study other classical thermally driven flows: the heated vertical plane plate and the Rayleigh-Bénard flow.

Concerning the thermally driven laminar flow along a vertical heated plate with isoflux condition, a self-similar solution is given by Sparrow and Gregg [3]. The wall temperature relative to the ambient and the thermal boundary layer thickness are found to be proportional to the distance to the leading edge to the power of 1/5, whereas velocity is found to follow a 3/5 power law. These power laws are easily retrieved by a scaling analysis as proposed by Lin and Armfield [4] that exhibits a steady state solution for Prandtl numbers ($Pr$) lower than unity. For Prandtl numbers larger than one, Lin, Armfield et al. [5] have developed a similar analysis for a vertical plate heated at a uniform temperature. In these studies, the viscous boundary layer is defined as the zone where vertical velocity is not negligible and it is split into inner- and outer-layers

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**Nomenclature**

| | |
|---|---|
| $b$ | Channel width (m) |
| $c_{fg}$ | Correlation coefficient for the quantities $f$ and $g$ |
| $f(x)$ | Function defined by Eq. (23) ($m^2\ s^{-2}$) |
| $g$ | Acceleration of gravity ($m\ s^{-2}$) |
| $H$ | Channel height (m) |
| $h'(x,y)$ | Function defined by Eqs. (19) and (20) ($m^2\ s^{-2}$) |
| $I_{u,entry}$ | Turbulent intensity at the entry |
| $l$ | Channel depth (m) |
| $P$ | Mean pressure (Pa) |
| $P_g$ | Mean driving pressure, $P_g = P + \rho_{inlet} g x$ (Pa) |
| $Q_{conv,\ i}$ | Vertical heat rate in Zone i per unit of depth ($W\ m^{-1}$) |
| $Q'_{conv,\ V}$ | Horizontal heat rate at the boundary V-VI per unit of depth ($W\ m^{-1}$) |
| $q_w$ | Wall heat flux ($W\ m^{-2}$) |
| $Ra^*$ | Modified Rayleigh number, $Ra^* = \frac{g\beta q_w b^5}{\lambda\nu\kappa H}$ |
| $T_c(x)$ | Mean temperature at the channel centre (K) |
| $T_{ref}$ | Reference temperature for the Boussinesq approximation (K) |
| $U,V,W$ | Mean velocity component in the directions $x,y,z$, respectively ($m\ s^{-1}$) |
| $u',v',w'$ | Instantaneous velocity fluctuations in the directions $x,y,z$, respectively ($m\ s^{-1}$) |
| $u^*,v^*,w^*$ | Order of magnitude of the velocity fluctuations in the directions $x,y,z$, respectively ($m\ s^{-1}$) |
| $U_{max}$, $U_{min}$ | Maximum and minimum of the mean vertical velocity profile, respectively ($m\ s^{-1}$) |
| $<u'v'>_{max}$ | Maximum value of the Reynolds stress at $x$ ($m^2\ s^{-2}$) |
| $x$ | Distance from the inlet in the ascendant direction (m) |
| $X$ | Order of magnitude of the distance from the inlet (m) |
| $y$ | Distance from the left wall (m) |
| $y_{SL}$ | Location of the right boundary of the shear layer at $x$, Eq. (33) (m) |
| $y_i$ | Right boundary location of Zone $i$ at $x$, see Table 1 (m) |
| $z$ | Distance from the front lateral wall (m) |
| $\alpha(x)$ | Part of heat rate extracted from Zone V, Eq. (8) |
| $\alpha'$ | Coefficient of the order of unity, Eq. (9) |
| $\beta$ | Isobaric thermal expansion of water ($K^{-1}$) |
| $\Gamma = \frac{H}{b}$ | Aspect ratio |
| $\delta_i$ | Order of magnitude of the thickness of Zone i at $x$, (m) |
| $\Delta T(x,y)$ | Mean temperature difference with the entry of the channel (K) |
| $\theta'$ | Instantaneous temperature fluctuations (K) |
| $\theta^*$ | Order of magnitude of the temperature fluctuations (K) |
| $\kappa$ | Thermal diffusivity of water ($m^2\ s^{-1}$) |
| $\lambda$ | Thermal conductivity of water ($W\ m^{-1}\ K^{-1}$) |
| $\nu$ | Cinematic viscosity of water ($m^2\ s^{-1}$) |
| $\rho$ | Density of water at the reference temperature ($kg\ m^{-3}$), |
| $\tilde{\rho}$ | Density of water ($kg\ m^{-3}$), |
| $< >$ | Time average operator |

*subscripts*

| | |
|---|---|
| $c$ | Referring to the centre of the channel |
| $i$ | Order of magnitude of a quantity in Zone $i \in \{I, II \ldots VII\}$ |
| inlet | Referring to channel inlet |
| surr | Referring to the surrounding of the channel |
| $t$ | Referring to the height of the transition |

with respect to the location of the velocity peak. To our knowledge, there has been no such scaling analysis for turbulent flow along a vertical plate. A similar analysis by Khanal and Lei [6] has been derived for a laminar thermally driven flow in a vertical channel. A vertical channel with one wall heated at a constant heat flux is analysed through a scaling analysis of the 2D-equations of conservation. In this study, the theory developed by Lin et Armfield [4] is adapted to the channel configuration. Two regimes are considered depending on whether the thermal boundary layers are distinct or not, the later case corresponding to very low Rayleigh numbers. For the distinct thermal boundary layers case and for the steady-state regime, the solution found by Lin and Armfield is retrieved. Hence this result is consistent with the analysis of Bar-Cohen and Rohsenow [7] that showed a good agreement of the experimental data by Sobel et al. [8] with the self-similar solution given by Sparrow and Gregg [3].

By contrast, a lot of scaling analysis have been developed for turbulent convection in the Rayleigh-Bénard configuration (see for example [9,10]). In particular, Grossmann and Lohse [11,12] have proposed a theory based on scaling analysis to obtain relationship between dimensionless numbers. Applied to Rayleigh-Bénard configuration, the so-called GL-theory considers the viscous and thermal turbulent dissipation rates in the whole flow as the sum of dissipation rates evaluated in the thermal boundary layer and in the bulk flow. It results in relationships between Nusselt, Rayleigh and Reynolds numbers that are the sum of two power laws. In these studies, the thermal boundary layer that is defined by the wall-normal temperature gradient, is supposed to be laminar. The scales of the thermal- and viscous-boundary layer thickness follow the classical Blasius law meaning that they are proportional to the Reynolds number to the power of $-1/2$, the Reynolds number being based on the height of the cavity and on the wind velocity [11]. This Blasius law also applied to the boundary layers thickness of the laminar flow in the vertical plate configuration [5]. Indeed, in the energy equation the balance between the vertical transport of heat by convection and the horizontal conduction of heat leads directly to the Blasius scaling law. Recently, Ng et al. [13] applied the GL-theory to thermally driven flow in a 2D vertical channel heated on one plate and cooled on the other one. The temperature of the plates are uniform and the channel is considered as infinitely long. Using 2D direct numerical simulation, they showed that the velocity and temperature scales has to be different in the boundary layers and in the bulk flow to verify the GL-theory. The determination of relationships between these scales has to be found out in order to obtain global Nusselt Rayleigh numbers relationship.

In this study, a scaling analysis of a thermally driven flow in a vertical channel with symmetrically heated plates at a constant heat flux is carried out. Considering that the height of the channel is much larger than its width and depth, the flow is studied through 2D boundary layer like equations. The analysis is based on the partition of the half-channel in 7 zones, each zone being characterized by specific scales. The hypothesis are verified by the comparison with experimental data reported in the companion paper. These experimental data correspond to a modified Rayleigh number: $Ra^* = \frac{g\beta q_w b^5}{\lambda\nu\kappa H} = 6.7 \times 10^7$ (for notations see the nomenclature). The article is organized as follows: the governing equations are given in Section 2 and the partition of the domain is described in Section 3. In Section 4, a zonal analysis is conducted. The flow is analysed in terms of heat transfer, pressure and Reynolds stress evolution along the channel. Some results are discussed in Section 5.